# Eurocode 3 and the in-plane stability of portal frames


## Synopsis

**Simple design rules are proposed that will enable engineers to take into account in-plane stability when designing single-storey steel portal frames plastically to Eurocode 3, without the need to resort to second-order elastic-plastic analysis software. The proposed design rules, developed from the results of a parametric study of different types of frame, are based on the Merchant-Rankine reduction method and take into account a modest amount of benefit from strain-hardening. A simple hand method for estimating the elastic critical load, required for calculating the Merchant-Rankine reduction, is also presented. From the results of the parametric study, the proposed design rules place frames into one of two categories:**

- **Category A: Regular, symmetric and mono-pitched frames**
- **Category B: Frames that fall outside of Category A but excluding tied portals**

**For each category of frame, a reduction factor based on the Merchant-Rankine reduction method is proposed.**


## Notation

$E$ Young's modulus
$G$ permanent load (dead load in UK practice)
$H$ horizontal reaction at column base
$h$ height of column
$H_{EHF}$ equivalent horizontal force at column top (see Fig 8(c))
$I_R$ second moment of area of rafters
$k$ strain hardening factor (determined from tests)
L span of frame
$N_{R,ULS}$ axial load in rafter at ULS calculated from first- order plastic analysis (see Fig 8 (b))
$Q$ variable load (live load in UK practice)
$V$ vertical reaction at column base
$V_{ULS}$ factored vertical reaction at ULS calculated from first-order plastic analysis (see Fig 8(b))
$\alpha_{p1}$ load factor at plastic collapse in first-order plastic analysis
$\alpha_{p2}$ load factor at plastic collapse in second-order elastic-plastic analysis
$\alpha_{cr}$ elastic critical buckling factor (calculated exactly)
$\alpha_{cr,est}$ $\alpha_{cr}$ estimated for the first sway mode
$\alpha_{cr,H}$ $\alpha_{cr}$ estimated by Horne
$\delta_{EHF}$ horizontal deflection of column top (see Fig 8(c))
$\gamma_G$ partial factor for permanent loads
$\gamma_P$ partial factor for pre-stressing forces
$\gamma_Q$ partial factor for variable loads
$\xi$ reduction factor on permanent loads
$\psi_i$ 'combination factor' reducing the $i$th variable load

## Introduction

Single-storey pitched roof steel portal frames are a very economical and popular form of structure, widely used for industrial and retail purposes. In the UK, such structures account for 90% of single-storey buildings and about 50% of all the steel used in construction.

Engineers generally achieve maximum economy in the design of single-storey steel portal frames through the use of plastic design. While in BS 449[1] the use of plastic design was permitted by a single clause, the trend towards lighter structures and more slender members has meant that the more modern codes of practice have been required to be more rigorous and take into account, amongst other things, in-plane frame instability.

BS 5950: Part 1: 1985 and BS 5950: Part 1: 1990[2, 3] included rules giving limits of sway stiffness such that global stability and second-order effects could be ignored. As a result of these rules, portal frames could continue to be designed plastically, without the need to resort to second-order elastic-plastic analysis. These rules, however, were later shown to be over-optimistic and unreliable for certain types and shapes of frame[4, 5]. The rules were subsequently revised in BS 5950-1: 2000[6] in order to avoid unsafe designs but this resulted in a reduction in economy for certain frames.

The forthcoming Eurocode 3 (EC 3)[7], will supersede the current national codes of practice. However, unlike BS 5950: 2000, there are no simple methods given for plastic design of portal frames to avoid the need for second-order elastic-plastic analysis.

Furthermore, the load combinations prescribed in BS EN 1990[8] differ from those in BS 5950-1: 2000. In addition to different partial load factors, the critical load combination for portal frames designed in accordance to EC3 normally includes a lateral wind load component; under BS 5950-1: 2000, the critical load combination for portal frames is usually only vertical load. For this reason, the BS 5950-1: 2000 design rules concerning global stability and second-order effects cannot be applied directly to design in accordance with EC 3.

This paper presents non-contradictory and complementary information for the plastic design of portal frames in accordance with EC 3. The BS EN 1990 load combinations are presented, and the beneficial effect of strain-hardening explained. A parametric study of different types of frame is then described. From the results of the parametric study, design rules are presented, based on the Merchant-Rankine reduction method, that will allow the majority of portal frames to be designed plastically without the need to resort to second-order elastic-plastic analysis software.

## Reduction in strength due to second-order effects

Fig. 1(a) shows a simply supported beam under a uniformly distributed load ($w$). From first-order elastic theory, the central deflection of the beam can be estimated accurately (ignoring shear deflection) from $5wL^4/384EI$. Fig 1(b) shows the same elastic beam under a small additional axial load ($P$); intuitively, the effect of such an axial load will be a small increase in the central deflection of the beam. However, as the axial load is increased further, the central deflection of the elastic beam will increase exponentially until failure occurs through instability due to buckling. The value of the axial load that will cause failure will depend on the magnitude of the uniformly distributed load; an upper bound to the value of the axial load will be the Euler strut buckling load $\pi^2EI/L^2$ (Fig 1(c)).

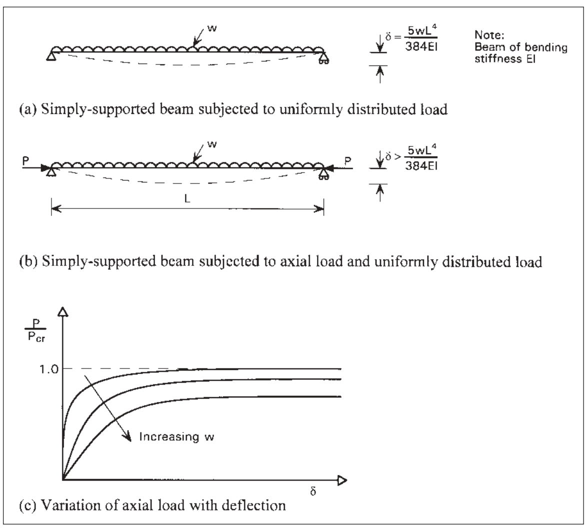


**Fig 1. Diagram illustrating sensitivity of axially loaded beams to second-order effects**


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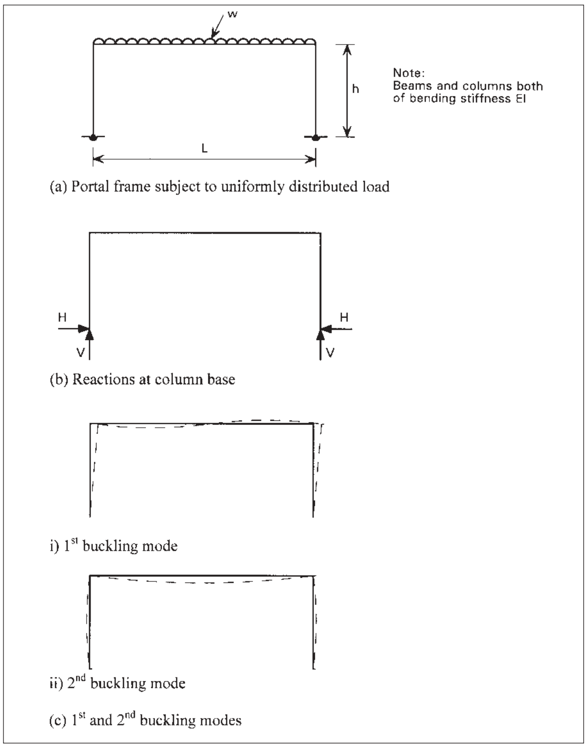


**Fig 2. Diagram illustrating sensitivity of portal frames to second-order effects**

The above example illustrates that the effect of axial load on a beam will be to increase deflections (and therefore bending moments, stresses and strains) beyond those calculated from first-order elastic theory. In order to predict accurately deflections of axially loaded beams, such as that shown in Fig 1(b), a second-order theory, which takes into account the destabilising effect of axial compressive loads together with finite deflections, will need to be applied. Axially loaded beams can therefore be seen to be sensitive to second-order effects, the degree of sensitivity in the elastic range being dependent on the ratio of the axial load to the Euler strut buckling load.

Fig 2(a) shows the case of a portal frame with members of uniform section loaded by a uniformly distributed load ($w$). This load induces both vertical reactions ($V$) and horizontal reactions ($H$) at the column bases (Fig 2(b)). The ratio $H/V$ is dependent on the span-to-height ratio $L/h$ of the frame, and is given by

$$\frac{H}{V} = \frac{(L/h)^2}{4 + 6(L/h)}$$

It can be seen that for a frame having an $L/h$ ratio of one, the horizontal reaction is only one tenth of the vertical reaction. On the other hand, if the $L/h$ ratio is as large as seven, the horizontal reaction, and consequently the axial compressive force in the rafter, is approximately equal to the vertical reaction.

As a result of the axial forces in the rafters and columns, portal frames can be susceptible to in-plane buckling; the first and second buckling modes of a portal frame under vertical loading are shown in Fig 2(c). Thus, as in the case of axially loaded beams, portal frames are sensitive to second-order effects, the degree of sensitivity in the elastic range being dependent on the ratio of the applied load to the load which causes elastic critical buckling of the frame.

It was explained in the introduction that economy in the design of steel portal frames is obtained by using plastic theory. However, portal frames designed using first-order plastic theory will not, by definition, take into account second-order effects. Consequently, first-order plastic theory will tend to overestimate the load at which sufficient plastic hinges form in the frame to give rise to a collapse mechanism. A designer therefore has to determine whether or not the reduction in load capacity due to second-order effects is small enough to be ignored. Here, the sensitivity is related to the ratio of elastic critical buckling load to the plastic collapse load of the frame. If the effects are small enough to be ignored, first-order plastic theory can be used for design. On the other hand, if the effects are not small enough to be ignored, they need to be taken into account in design[9]. The formal method of taking into account the influence of second-order effects on the plastic collapse of steel frames is by means of a second-order elastic-plastic analysis which traces the successive formation of plastic hinges as the load is increased. This requires sophisticated software which may not be necessary in many cases. This paper presents a greatly simplified method for taking into account second-order effects in design in accordance with EC 3.

## Beneficial effect of strain-hardening

There are two distinct benefits which may arise from the presence of strain hardening in the formation of a plastic hinge. The first benefit is that in a region of approximately constant bending moment, the moment of resistance may rise to up to 8% above the calculated value of the fully plastic moment ($M_p$) and then stays approximately constant as the plastic hinge rotates[10]. Though beneficial, this effect is conservatively ignored in the following sections of this paper.

The second benefit is that when a plastic hinge forms in a region of significant bending moment gradient, it initially forms at the calculated nominal value of the fully plastic moment ($M_p$) and then rises steadily as this hinge rotates during the elastic-plastic stage of loading to collapse. When conducting a frame analysis, taking into account the effect of strain-hardening in this way will result in an increased plastic collapse load of the frame and this helps to offset the reduction in strength due to second-order effects. It is implicit that this may result in a modest increase in the bending moments at some of the connections, notably at the eaves, and this should be borne in mind when carrying out the detailed design of these connections.

The relationship between plastic hinge rotation ($\phi$) and increased bending moment due to strain hardening ($dM$) has been expressed by Davies[11, 12] in terms of a numerical strain-hardening factor ($k$)

$$k = \left(\frac{\phi}{dM}\right)\left(\frac{EI}{h^*}\right)$$

where

$\phi$ rotation of the hinge
$dM$ increase in bending moment above $M_p$
$h^*$ length related to the ratio of plastic moment to shear force at the hinge (see Ref 11 and Ref 12 for a more detailed explanation).

From the above equation it can be seen that the higher the value of $k$, the smaller the increase in moment resistance of the hinge ($dM$) due to strain-hardening. The assumed strain-hardening factor embodied in design rules is around 10 to 12, as exemplified in the BCSA 'Black Book' No 29[13]. This is justified by a recent study[14] utilising numerical integration of stress-strain data from mill tests which showed that, neglecting any local or lateral-torsional buckling, for both S275 and S355 steel, and for a wide range of section sizes, $k$ is close to 10.

However, a recent review of test results[15] indicates that a more realistic value of k in typical I-sections may be about 20. Current work by Davies at Manchester[12], also indicates that the value of $k$ may indeed be greater than 10, the reduced benefit of strain-hardening being attributed to interaction between local instability of the compression flange at the plastic hinge and lateral torsional buckling of the adjacent member. In the parametric study described in later on in this paper, a value of $k$ of 20 has been adopted.

## Frame loading

### *Load combinations to BS EN1990*

In accordance to EC 3, the load combinations for steel structures are defined in BS EN 1990[8] as either

$$\sum_{j \geq 1}\gamma_{G,j}G_{k,j} \text{ "+" } \gamma_p P \text{ "+" } \gamma_{Q,1}Q_{k,1} \text{ "+" } \sum_{i > 1}\gamma_{Q,i}\psi_{0,i}Q_{k,i}$$

...(eq 6.10)

or the less favourable of the two following expressions

$$\sum_{j \ge 1}\gamma_{G,j}G_{k,j}\ \text{"+"}\ \gamma_p P\ \text{"+"}\ \gamma_{Q,1}\psi_{0,1}Q_{k,1}\ \text{"+"}\ \sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i}$$

(eq 6.10a)

$$\sum_{j \ge 1}\xi_j\gamma_{G,j}G_{k,j}\ \text{"+"}\ \gamma_p P\ \text{"+"}\ \gamma_{Q,1}Q_{k,1}\ \text{"+"}\ \sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i}$$

(eq 6.10b)

As equations 6.10a and 6.10b will result in lighter loads than equation 6.10, equations 6.10a and 6.10b were applied in the parametric study.

Using the recommended values of $\xi$, $\gamma_G$, $\gamma_Q$, $\psi_0$ (slightly different values may be recommended by the UK National Annex) the following four load combinations (LC) were considered:

1.15 Dead + 1.50 Live + 0.75 Wind + NHL (LC1)
1.15 Dead + 0.75 Live + 1.50 Wind + NHL (LC2)
1.35 Dead + 0.75 Live + 0.75 Wind + NHL (LC3)
1.15 Dead + 1.50 Live + NHL (LC4)

The notional horizontal load (NHL), applied horizontally to the top of each column, is taken as 0.5% of the factored reaction at the base of the column, the basic value given in EN 1993-1-1.

LC1, LC2 and LC3 all include a lateral wind load component. However, from inspection, it can be seen that LC3 (in which only the dead load component is higher than those of LC1 and LC2) will result in the lightest loads. Only LC1 and LC2 were applied in the parametric study.

Load combination 4, comprising only vertical load, has the same partial load factors as LC1. On a long span single-storey building, LC4 could be critical as the wind load may result in a significant uplift, reducing the total vertical load. However, for the purposes of the parametric study, LC4 was ignored because uplift reduces the compression forces thus reducing second-order effects.

It should be noted that under BS 5950-1: 2000, the critical load combination is usually the vertical load combination:
1.4 Dead + 1.6 Live + NHL

As explained in the introduction, the presence of the wind load (giving lateral loads) in the EC3 load combinations means that the rules for taking into account second-order effects in BS 5950: 2000 should not be applied to design in accordance to EC 3.

### *Dead and live loads*

In the parametric study, the following dead and live loads were applied to the frames:
Dead load: 0.15 kN/m$^2$ + self-weight of frame
Live load: 0.6 kN/m$^2$

### *Wind load*

The wind load adopted for the parametric study assumes that the portal frame will be subject to the average wind speed in the U.K. of 23.5m/s, but that the portal frame will be situated 0m from the sea. The effect of such a combination is a wind load intensity 40% higher than that of a portal frame designed for a wind speed of 23.5m/s and situated 100km from the sea, or 5% higher than that of a portal frame designed for a wind speed of 24m/s and situated 5km from the sea.

As the portal frames considered in the parametric study will be subjected to a high wind load intensity, the design rules proposed can be considered as being conservative.

### *Parametric study*

Parametric studies were conducted using a prototype analysis engine that can take into account both second-order effects and strain-hardening, similar to that described in Reference 16. The geometry and loading were set up for the analysis using the portal frame design program CSC Fastrak. In total, 20 frames were analysed as part of the parametric study, covering a large range of geometry and types of frame.

### *Frame geometry*

The frames considered as part of the parametric study are shown in Table 1. Regular frames have symmetrical bays with rafters of uniform slope and columns of equal length. Regular multi-span frames have a series of identical spans. The column bases of all of the frames were taken as pinned. The length and depth of the eaves haunch were assumed to be span/10 and span/50, respectively; similarly, the length and depth of the apex haunch were assumed to be span/20 and span/75, respectively. A bay spacing of 6m was adopted.

### *Member sizing*

Using the prototype analysis engine each frame was designed, and its members sized, taking into account both second-order effects and strain-hardening. As discussed in a previous section, only load combinations 1 and 2 were considered. When designing each frame, the members were sized so that the critical second-order elastic-plastic collapse load factor of the frame, $\alpha_{p2}$, would be as close to unity as possible without the frame failing. It should be noted that only in-plane stability was considered, and not out-of-plane stability of the lengths of frame member between adjacent purlins and side rails. As universal beam section sizes are discrete, obtaining a value of $\alpha_{p2}$ close to unity was not always easy to achieve.

### *Analysis results*

For each of the frames designed, and for each of the two load

*Table 1: Frames considered as part of parametric study*

| Frame | Number of spans | Frame category[1] | L/h[2] | | | Pitch | Description |
|---|---|---|---|---|---|---|---|
| 1 | 1 | A | 8 | — | – | 6° | Single span |
| 2 | 1 | A | 5 | – | – | 6° | Single span |
| 3 | 2 | A | 2 | – | - | 6° | Twin span |
| 4 | 2 | A | 5 | - | - | 6° | Twin span |
| 5 | 3 | A | 8 | – | – | 6° | Three span |
| 6 | 3 | A | 5 | – | - | 6° | Three span |
| 7 | 3 | A | 2 | – | – | 6° | Three span |
| 8 | 6 | A | 5 | – | – | 6° | Six span |
| 9 | 1 | A | 5 | – | – | 30° | Steep single span |
| 10 | 1 | B | 5 | – | – | 65° | Mansard |
| 11 | 1 | B | 5 | – | – | 60° | Pseudo curved |
| 12 | 2 | B | 8 | 5 | – | 6° | Varied span |
| 13 | 3 | B | 8 | 5 | 5 | 6° | Varied span |
| 14 | 1 | B | 5 | – | — | 6° | Mezzanine |
| 15 | 2 | B | 5 | – | – | 6° | Mezzanine |
| 16 | 2 | B | 5 | – | – | 6° | Varied height twin span |
| 17 | 3 | B | 5 | 5 | 2 | 6° | Varied height three span |
| 18 | 2 | B | 5 | – | – | 30° | Flat-topped |
| 19 | 1 | excluded | 5 | – | – | 6° | Tied portal |
| 20 | 2 | excluded | 5 | – | – | 6° | Tied portal |

[1] Frame Category defined in 'Proposed design rules' section
[2] L/h for other spans only given if different from first span

*Table 2: Reduction factor of frames for load combination 1*

| Frame | $\alpha_{cr,norm}$ | $\alpha_{p2}/\alpha_{p1}$ | $(\alpha_{p2}/\alpha_{p1})_{MR}$ | $(\alpha_{p2}/\alpha_{p1})/(\alpha_{p2}/\alpha_{p1})_{MR}$ |
|---|---|---|---|---|
| 1 | 5.66 | 0.96 | 0.82 | 1.17 |
| 2 | 7.03 | 0.95 | 0.86 | 1.11 |
| 3 | 6.92 | 0.88 | 0.86 | 1.03 |
| 4 | 4.18 | 0.89 | 0.76 | 1.17 |
| 5 | 2.63 | 0.88 | 0.62 | 1.42 |
| 6 | 3.49 | 0.90 | 0.71 | 1.26 |
| 7 | 2.76 | 0.77 | 0.64 | 1.21 |
| 8 | 2.11 | 0.86 | 0.53 | 1.63 |
| 9 | 7.39 | 0.90 | 0.86 | 1.04 |
| 10 | 4.84 | 0.75 | 0.79 | 0.95 |
| 11 | 6.67 | 0.78 | 0.85 | 0.92 |
| 12 | 7.67 | 0.85 | 0.87 | 0.98 |
| 13 | 5.48 | 0.82 | 0.82 | 1.00 |
| 14 | 3.75 | 0.82 | 0.73 | 1.12 |
| 15 | 3.54 | 0.78 | 0.72 | 1.09 |
| 16 | 5.54 | 0.93 | 0.82 | 1.13 |
| 17 | 5.03 | 0.93 | 0.80 | 1.16 |
| 18 | 5.17 | 0.88 | 0.81 | 1.09 |
| 19 | 1.93 | 0.57 | 0.48 | 1.18 |
| 20 | 2.02 | 0.63 | 0.50 | 1.25 |

*Table 3: Reduction factor of frames for load combination 2*

| Frame | $\alpha_{cr,norm}$ | $\alpha_{p2}/\alpha_{p1}$ | $(\alpha_{p2}/\alpha_{p1})_{MR}$ | $(\alpha_{p2}/\alpha_{p1})$ $/(\alpha_{p2}/\alpha_{p1})_{MR}$ |
|---|---|---|---|---|
| 1 | 6.46 | 0.90 | 0.85 | 1.06 |
| 2 | 8.03 | 0.93 | 0.8 | 1.06 |
| 3 | 10.94 | 0.90 | 0.91 | 0.99 |
| 4 | 5.61 | 0.84 | 0.82 | 1.02 |
| 5 | 3.54 | 0.73 | 0.72 | 1.02 |
| 6 | 4.36 | 0.80 | 0.77 | 1.04 |
| 7 | 4.45 | 0.75 | 0.78 | 0.97 |
| 8 | 2.64 | 0.74 | 0.62 | 1.19 |
| 9 | 9.87 | 0.94 | 0.90 | 1.05 |
| 10 | 7.15 | 0.84 | 0.86 | 0.98 |
| 11 | 9.78 | 0.89 | 0.90 | 0.99 |
| 12 | 7.02 | 0.94 | 0.86 | 1.10 |
| 13 | 5.38 | 0.87 | 0.81 | 1.07 |
| 14 | 5.28 | 0.85 | 0.81 | 1.05 |
| 15 | 4.38 | 0.80 | 0.77 | 1.04 |
| 16 | 8.61 | 0.90 | 0.88 | 1.02 |
| 17 | 7.69 | 0.82 | 0.87 | 0.94 |
| 18 | 7.39 | 0.76 | 0.86 | 0.88 |
| 19 | 1.11 | 0.56 | 0.10 | 5.65 |
| 20 | 2.83 | 0.60 | 0.65 | 0.93 |

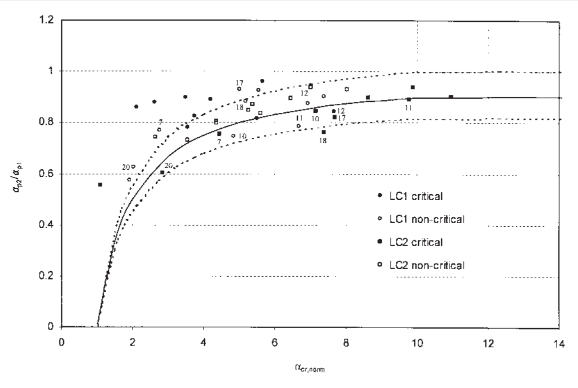


**Fig 3. Plot showing reduction factor for all frames**

combinations, two collapse load factors were determined:

- Second-order elastic-plastic with strain-hardening ($\alpha_{p2}$)
- First-order plastic (without strain-hardening) ($\alpha_{p1}$)

The reduction factor $\alpha_{p2}/\alpha_{p1}$ was then calculated for each load combination.

In addition, for each frame and load combination, Fastrak was used to determine the elastic-critical load factor, $\alpha_{cr}$. A normalised value of the elastic-critical load factor, $\alpha_{cr,\ norm}$ was calculated from

$$\alpha_{cr,\ norm} = \frac{\alpha_{cr}}{\alpha_{p2}}$$

This normalised value for the elastic-critical load factor corresponds to that of a frame having a value of $\alpha_{p2}$ of unity. Table 2 and Table 3 show these results for load combinations 1 and 2, respectively.

### *Merchant-Rankine*

Fig 3 shows the reduction factor $\alpha_{p2}/\alpha_{p1}$ plotted for each frame against the normalised elastic critical load factor $\alpha_{cr,\ norm}$. The load combination to which each result corresponds can be identified, as well as whether or not the load combination is critical.

Merchant[17, 18] proposed that the reduction factor from first-order plastic to second-order elastic-plastic may be related to

$$\left(\frac{\alpha_{p2}}{\alpha_{p1}}\right)_{MR} = \frac{\alpha_{cr} - 1}{\alpha_{cr}}$$

This expression is analogous to the 'Rankine' equation for predicting the failure load of a pin-ended strut. For this reason, it is generally known as the 'Merchant-Rankine' formula. In Table 2 and Table 3, the Merchant-Rankine reduction factor $(\alpha_{p2}/\alpha_{p1})_{MR}$ has been calculated. The ratio of the Merchant-Rankine reduction factor to the actual reduction factor $(\alpha_{p2}/\alpha_{p1})_{MR}$ has also been calculated.

The middle of the three curves shown in Fig 3 is the Merchant-Rankine reduction factor. The lower of the three curves corresponds to that of a reduced Merchant-Rankine defined as

$$\left(\frac{\alpha_{p2}}{\alpha_{p1}}\right)_{MR,\ red} = \frac{\alpha_{cr} - 1}{1.1\alpha_{cr}}$$

The upper of the three curves corresponds to that of an enhanced Merchant-Rankine defined as

$$\left(\frac{\alpha_{p2}}{\alpha_{p1}}\right)_{MR,\ enh} = \frac{\alpha_{cr} - 1}{0.9\alpha_{cr}}$$

Frames having a reduction factor lower than that predicted by Merchant-Rankine have been identified in Fig 3.

The design rules proposed later in this paper require use of both the Merchant-Rankine and the reduced Merchant-Rankine curves, but not of the enhanced Merchant-Rankine curve. Nevertheless, the enhanced Merchant-Rankine curve is shown in all the plots presented in this paper. The enhanced Merchant-Rankine curve is similar to a curve proposed by Wood[19]. Wood intended this curve to be used for frames in multi-storey buildings, but not for portal frames, when no direct account is taken of the stiffening effects of the cladding, partitions etc. Subsequently, it has appeared in BS 5950 and has been regarded as some justification for EC 3 allowing first-order analysis if $\alpha_{cr}$ is at least 10. The results presented in this paper confirm Wood's view, based on engineering judgement, that the enhanced Merchant-Rankine curve is too optimistic for portal frames.

### *Discussion of results*

*Regular single-span and multi-span frames*

The results for regular single-span and multi-span frames are shown in Fig 4. As can be seen, only Frame 7, a three span frame having an $L/h$ of 2, has a reduction factor 3% lower than that predicted by Merchant-Rankine.

Previously in this paper it was explained that frames having a low value of $L/h$ are less susceptible to second-order effects than frames having a high value of $L/h$. It may therefore seem surprising that Frame 7, having an $L/h$ of 2, has a reduction factor lower than that predicted by Merchant-Rankine while Frame 6, having an $L/h$ of 8, has a reduction factor higher than

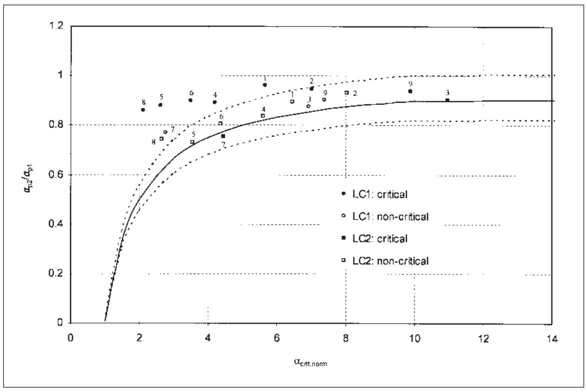


**Fig 4. Plot showing reduction factor for single and multi span frames (Frames 1-9)**

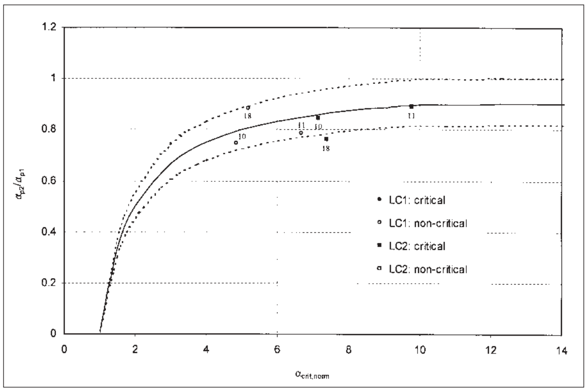


**Fig 5. Plot showing reduction factor for arched frames (Frames 10, 11 and 18)**

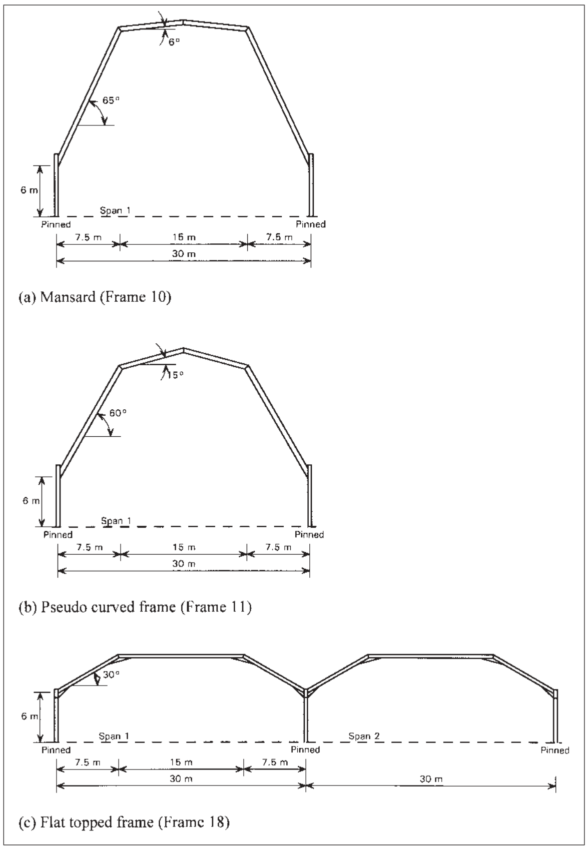


**Fig 6. Arched frames**

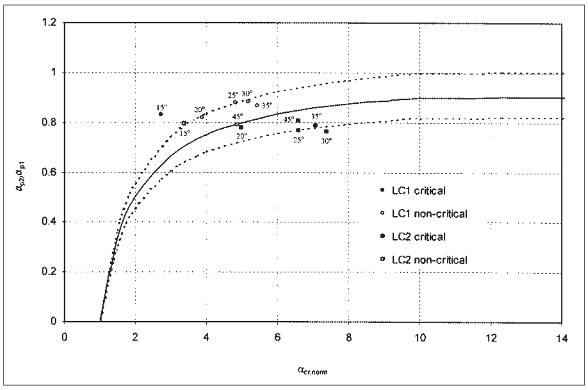


**Fig 7. (left) Plot showing reduction factor for two span flat topped frames having different pitches**

that predicted by Merchant-Rankine.

The results of Frames 6 and 7 may be explained by the fact that the hinge rotation of a frame having a low value of $L/h$ is less than that of a frame having a high value of $L/h$. Previously, it was explained that beneficial effect of strain-hardening can offset the reduction in strength due to second-order effects. The results of Frames 6 and 7 therefore show that frames having a low value of $L/h$ benefit less from strain-hardening than frames having a high value of $L/h$.

The fact that a single frame, Frame 7, has a reduction factor 3% lower than that predicted by Merchant-Rankine is considered to be statistically acceptable and will be ignored for the purposes of proposing design rules.

*Arched frames*

The results for the flat topped, Mansard and pseudo-curved frames are shown in Fig 5. Owing to their shape, these three frames will be referred to as arched frames (Fig 6). The arch shape of these frames means that the bending moment diagram closely follows the shape of the frame; as a result, such frames therefore have more slender rafter members than duo-pitch frames of similar geometry.

From Fig 5 it can be seen that the reduction factor $\alpha_{p2}/\alpha_{p1}$ for all three frames is below that predicted by Merchant-Rankine. In the case of Frame 18 the value of $\alpha_{p2}/\alpha_{p1}$ is 3% less than that predicted by the reduced Merchant-Rankine, defined in the previous section. This single result is not considered to be statistically significant in the design procedure which follows.

Frame 18 is a 2-span flat topped frame having a value of L/h of 5 and a pitch of 30°. A further parametric study was conducted in which the pitch of the 2-span flat topped frame was varied between 15° and 45°. The results of this parametric study are plotted in Fig 7. As can be seen, frames having a pitch of 30° have the lowest reduction factor $\alpha_{p2}/\alpha_{p1}$.

From interpolation of the results, if the pitch was approximately 18° then the value of $\alpha_{p2}/\alpha_{p1}$ would be equal to that predicted by Merchant-Rankine. Similarly, if the pitch of the frame was 25° then the value of $\alpha_{p2}/\alpha_{p1}$ would be equal to that predicted by the reduced Merchant-Rankine.

## Estimate of $\alpha_{cr}$

In the previous section, the Merchant-Rankine formula was used to predict the reduction factor $\alpha_{p2}/\alpha_{p1}$ from the elastic critical buckling load acr; the value of $\alpha_{cr}$ was calculated exactly using the Fastrak software. As the design rules proposed later in the paper will be based on the Merchant-Rankine formula, a method for estimating $\alpha_{cr}$, based on frame deflections, will be required.

Horne[20, 21, 22, 23] demonstrated that $\alpha_{cr}$ could be calculated sufficiently accurately for a for a multi-storey frame from

$$\alpha_{cr,H} = \left(\frac{h}{V_{uls}}\right)\left(\frac{H_{EHF}}{\delta_{EHF}}\right)$$

The parameters used to calculate $\alpha_{cr,H}$ for a portal frame are shown in Fig 8. As can be seen, $\delta_{HEF}$ is the lateral deflection at the top of each column when subjected to an arbitrary lateral load $H_{EHF}$.

It should be noted that although the magnitude of the lateral load is arbitrary (as it is simply used to calculate the sway stiffness $H_{EHF}/\delta_{EHF}$), the horizontal load applied at the top of each column should satisfy the following relationship

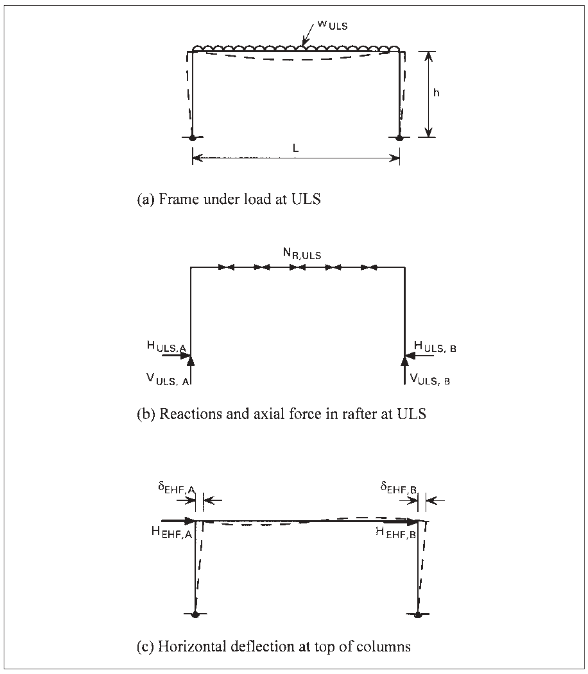


**Fig 8. (right) Diagram showing parameters required to estimate $\alpha_{cr}$**

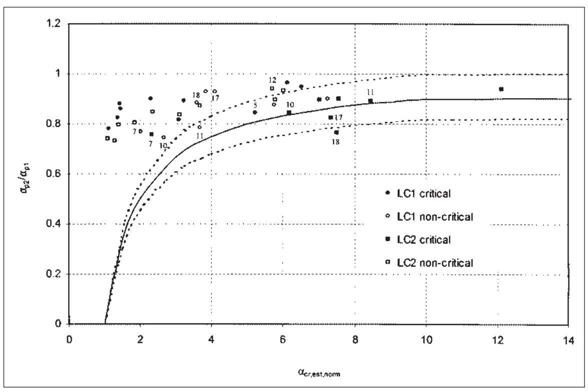


**Fig 9. Plot showing reduction factor using $\alpha_{cr,est}$ for all frames except tied portals**

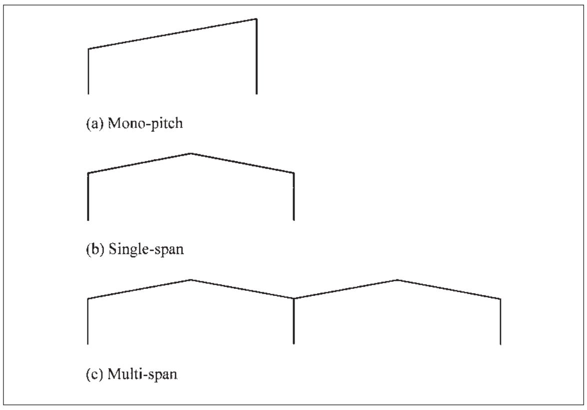


**Fig 10. Examples of Category A frames (a) Mono-pitch (b) Single-span (c) Multi-span**

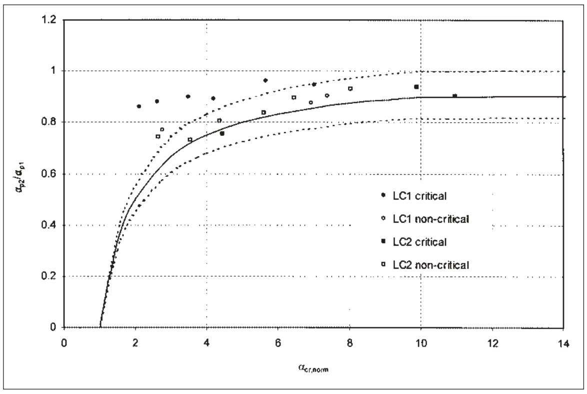


**Fig 11. Plot showing reduction factor for Category A frames**

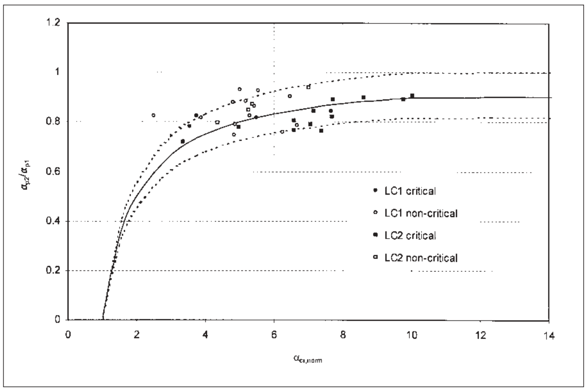


**Fig 12. Plot showing reduction factor for Category B frames**

$$\frac{H_{EHF}}{V_{ULS}} = \frac{H_{EHF,\,A}}{V_{ULS,\,A}} = \frac{H_{EHF,\,B}}{V_{ULS,\,B}}$$

Horne's method for estimating $\alpha_{cr}$ is unconservative when applied to portal frames i.e. overestimating the value of $\alpha_{cr}$. This is because Horne's method does not take into account the axial force in the rafters and also may underestimate the second-order effects in the columns. King[24] proposed the following modification to Horne's method, to take into account the axial force in the rafters and to increase the influence of second-order effects in the columns

$$\alpha_{cr,\,est} = 0.8\left\{1 - \left(\frac{N_{R,\,ULS}}{N_{R,\,cr}}\right)_{\max}\right\}\alpha_{cr,\,H}$$

where

$$N_{R,\,cr} = \frac{\pi^2 EI_R}{L^2}$$

Fig 9 shows the reduction factor $\alpha_{p2}/\alpha_{p1}$ plotted for each frame against $\alpha_{cr,\,est,\,norm}$, the normalised elastic critical load factor; the two tied portal frames have been excluded. The frames identified in Fig 3 as having a reduction factor lower than that predicted by Merchant-Rankine have also been identified in Fig 9.

As can be seen from Fig 9, if $\alpha_{cr}$ is calculated using King's estimate, only Frames 17 and 18 have a reduction factor lower than that proposed by Merchant-Rankine. In general, it can be seen that use of $\alpha_{est}$ results in a more conservative Merchant-Rankine reduction factor than use of the exact value of $\alpha_{cr}$ (calculated from software).

## Exclusion of certain types of frame from proposed design rules

It was explained previously that frames having high values of $L/h$ will be more susceptible to second-order effects. Frames having a value of $L/h$ greater than 8 are therefore excluded from the proposed design rules given below; such frames should be designed using second-order elastic-plastic analysis.

Tied portals are designed with low roof slopes and for least weight sections. Owing to the high axial force in the rafters, the non-linear behaviour of these frames is complex. As the Merchant-Rankine formula will not be able to take accurately into account the instability in tied-portals, such frames are therefore also excluded from the proposed design rules given in the following section. Tied portals should be designed using second-order elastic-plastic analysis; it should be noted that BS 5950: 2000 also requires second-order elastic-plastic analysis for tied portals.

## Proposed design rules

The proposed design rules are based on the Merchant-Rankine reduction method and exclude the following

- Frames in which $\frac{L}{h} > 8$ for any span
- Frames in which $\alpha_{cr} \leq 3$
- Tied portals

Frames excluded from the proposed design rules should be designed plastically using second-order elastic-plastic analysis software.

### *Category A: Regular, symmetric and asymmetric pitched and mono-pitched frames*

Regular, symmetric and mono-pitched frames (Fig 10) are either single-span frames or multi-span frames in which there is only a small variation in height ($h$) and span ($L$) between the different spans; variations in height and span of the order of 10% may be considered as being sufficiently small. For such frames, the second-order elastic-plastic collapse factor, $\alpha_{p2}$, may be calculated using the Merchant-Rankine formula

$$\alpha_{p2} = \alpha_{p1}\left(\frac{\alpha_{cr} - 1}{\alpha_{cr}}\right)$$

Table 1 identifies Category A frames. As an example, Frame 9, the 30° single span frame, is classified as a Category A frame. The results for all Category A frames described in this paper are shown in Fig 11.

### *Category B: Frames that fall outside of Category A and excluding tied portals*

For frames that fall outside of Category A and are not tied portals, the second-order elastic-plastic collapse factor, $\alpha_{p2}$, may be calculated from the reduced Merchant-Rankine expression, defined previously

$$\alpha_{p2} = \alpha_{p1}\left(\frac{\alpha_{cr} - 1}{1.1\alpha_{cr}}\right)$$

The results for all Category B frames described in this paper are shown in Fig 12; the reduced Merchant-Rankine curve is the dashed curve shown below the Merchant-Rankine curve. It should be noted that the Mezzanine frames and the varied height frames fall within Category B frames.

## Conclusions

- The parametric study has shown that for many portal frames the second-order elastic-plastic collapse factor can safely be predicted using either Merchant-Rankine or a reduced Merchant-Rankine, applied to first-order plastic analysis.
- The application of Merchant-Rankine requires a reasonable estimate of the elastic critical buckling load factor. A conservative method is proposed in this paper that avoids the necessity to use software to determine this load factor.
- Categories of frame are defined to which the Merchant-

Rankine or reduced Merchant-Rankine approach can be applied – designated 'Category A' and 'Category B'.

- Certain frames fall outside of these categories e.g. tied portals and for these frames second-order elastic-plastic analysis is required.

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