Global analysis: Back to the future

Dr Paul Howson, senior lecturer at Cardiff School of Engineering and member of IStructE Research Panel, writes on the benefits of using simple global models of structures in the early stages of the design process

The ability to model complex, three-dimensional, multi-bay, multi-storey structures to a high degree of accuracy has become commonplace over the last 20 years due to the widespread availability of powerful desktop computers and a variety of inexpensive finite element software. The resulting models are often referred to as 'global' or 'holistic' since they model the whole structure in its entirety. Thus any interaction between structural components such as frames, shear walls, cores etc. or coupling due to asymmetry of the plan form are automatically accounted for.

Such complex models offer considerable insight into the behaviour of the physical structure and their use for detailed design and analysis is not in question. However, the development of such a model at an early stage in a design process can be time consuming and unproductive if used during a period of rapid evolution of the concept. A compelling alternative is to use a simpler model, developed especially for the type of structure under consideration, which models only the dominant characteristics of the structure. This simplified global model can offer a number of potential benefits. Data preparation and analysis will be quicker; the modelling procedures are likely to be simpler and more transparent; the accuracy will normally be sufficient for preliminary assessment or checks on solutions obtained elsewhere; it can be used efficiently to improve and develop a feel for structure; it draws directly on the engineer's experience and judgement and it's use is an inclusive process, where the engineer is at the heart of the solution procedure in a way that can sometimes appear to be lost when using fully automated, general software[1].

Some background to these ideas is given in the remainder of this article, together with the arguments that enable a simplified global model to be developed for a plane frame. This example is the starting point for a particular family of such models that, in their most demanding form, can analyse the static, dynamic and buckling characteristics of three-dimensional buildings with doubly asymmetric plan form, varying properties at each floor level and containing a mix of structural frames, shear walls and cores.

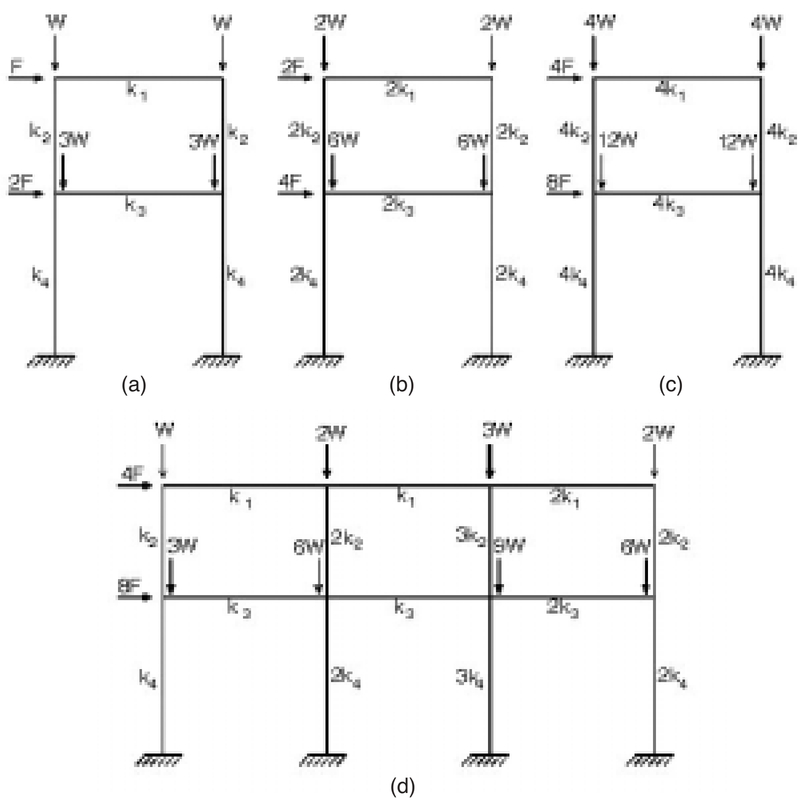


Fig 1. Frames conforming to the Principle of Multiples

Prior to the 1960s, the teaching and practice of structural engineering consisted mainly of understanding the underlying principles, learning hand methods and then practising their use extensively. The resulting models were often quite crude, but yielded conservative results that could be used directly in final designs etc. Because hand solutions were tedious, engineers thought particularly carefully about each problem, both before analysing it and as the analysis gave intermediate results that were expected or otherwise. Senior designers developed much of their experience by performing such calculations for each design they were responsible for, often including the analysis of several alternative structures before the design process was completed. This enabled them to develop insights that would allow a good design to be chosen prior to calculations commencing and/or to proceed from rejected designs to acceptable ones via as few analyses of trial designs as possible.

Over the intervening years, the methods have changed but the goals have not; namely to reach a satisfactory design in the least time, at minimum cost and with some educational benefit to the engineer. An attractive way of achieving these goals is to use the simplified global models in an analogous way to the hand techniques. However, there are a number of differences. The new models can be solved quickly and efficiently using modern computer techniques and their accuracy will normally be superior. However, accuracy is now less likely to be an issue, since the final design will normally be analysed using a more complex model. This offers great potential for a variety of simplified global models to be developed that can explore trends, establish design sensitivity and provide structural insights and experience.

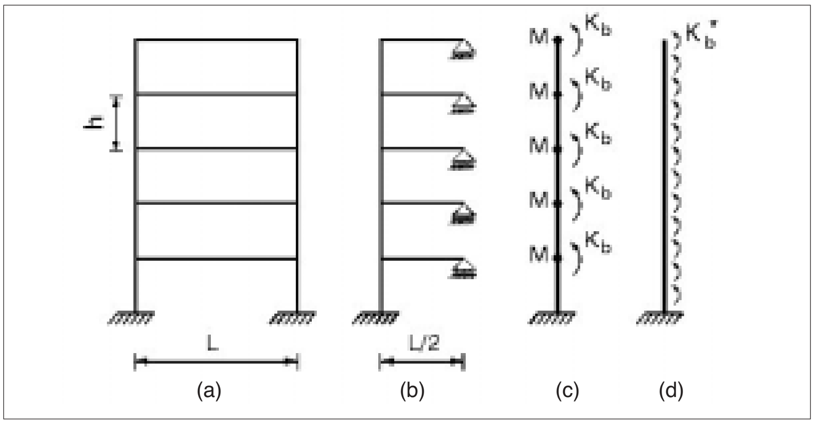
As an example, consider the development of a simplified global model that describes the lateral displacement of a plane frame. The approach depends on the Principle of Multiples, which was widely used when developing hand methods, but is now rarely used, like so many older techniques, in the mistaken belief that they have little or no value in modern processes.

The Principle of Multiples applies to unbraced, rigidly jointed, multi-bay, multi-storey plane frames and is exact on the basis of inextensible member theory. Fig 1 applies to deflection calculations due to lateral load F. It is usual to perform the lateral load calculations with $W = 0$, but non-zero values of W can be used if the designer wishes to allow for the magnifying effect that vertical loads have on horizontal deflections caused by lateral loading. The Principle of Multiples proves that the frames of Figs 1(a)-(d) share the same horizontal deflections for lateral loading problems. The reasons are as follows.

In Fig 1, the ks are values of EI/L for the members, where EI is the flexural rigidity and L is the length. Additionally, values are identical when the subscripts are identical, so that the frame of Fig 1(a) is symmetrical. Note also that the vertical loading is symmetric and the lateral loading is anti-symmetric i.e. the F of Fig 1(a) can be replaced by F/2 at the left hand end of the top team and F/2 at the right hand end of the top beam etc. Therefore the frame will sway with an anti-symmetric deflection pattern. Hence any frame which is identical to frame (a) must have the same deflected shape. Therefore any frame obtained by superposing N (which need not be integer) such frames, in the sense implied by frames (b) and (c), must also share the deflected shape of frame (a), even if the frames are all clamped together. Hence putting N = 2 and N = 4 gives the required proofs for frames (b) and (c), respectively. Moreover, frame (d) can be obtained by fastening together two frame (a)s and a frame (b), which are situated side by side in the appropriate way. Since frames (a) and (b) share the same anti-symmetric deflection pattern, the process of fastening them together to form frame (d) leaves the deflections unaltered.

Naturally, most practical multi-bay, multi-storey, plane frames do not obey the Principle of Multiples due to irregular bay widths, storey heights etc. However, a well established method, combined with engineering judgement, enables them to be reduced to an approximate single bay, multi-storey 'substitute' frame that can then be used to obtain approximate lateral deflection results for the multi-bay case. The substitute frame has the same number of storeys and the same storey heights as the actual frame, but differs in that it has only one bay, is symmetric and carries symmetric vertical loads. For a regular frame, the required details of the substitute frame are found from the actual frame as follows: the substitute column k is equal to half the sum of the ks for all actual columns at the same storey level; the substitute beam k is equal to the sum of the ks for all beams at the same storey level; the horizontal loads at the nodes at both ends of a beam are equal to half of the sum of the horizontal loads at all actual nodes at that storey level; and the values of W for the substitute columns are equal to half the sum of the axial forces in all actual columns at the same storey level. Applying the above rules to the frame of Fig 1(d) gives the frame of Fig 1(c), on which the forces 4F and 8F can be replaced by anti-symmetrical pairs of forces. Hence it can be deduced that when a frame obeys the Principle of Multiples the rules yield a substitute frame which gives exactly correct results for the actual frame, remembering that inextensible member theory is assumed. If it does not, the substitute frame will be approximate and the degree of approximation will depend on the irregularity of the original frame.

Fig 2. Simplification of the substitute frame



Since the substitute frame of Fig 2(a) is symmetric and only the anti-symmetric displacement is of interest, the substitute frame can be replaced by the model of Fig 2(b). This can be further simplified by replacing it with a bending cantilever that carries a lumped mass and a rotational spring stiffness at each floor level to represent the beams and any additional imposed loads that they may carry, see Fig 2(c). Assuming inextensible member theory, the required static stiffness is easily determined from the slope deflection equations to be $K_b = 6k$, where $k=EI/L$ for a substitute beam. In order to minimise the number of nodes, the final simplification is to smear the weight and stiffness from the beams along the column so that they become uniformly distributed between the nodes. When consecutive storeys are identical, their smeared properties define a member, which together with other members form the final model. This leads to the uniform cantilever of Fig 2(d) when all storeys are identical. The model then corresponds to a 2 × 2 stiffness matrix developed from the original substitute frame column, but carrying the added distributed weight and stiffness of, respectively, $w^* = Mg / h$ and $K_b^* = K_b / h$, where g is the acceleration due to gravity and h is the storey height.

In this particular instance, the model is solved using the stiffness technique and allows for the case of infinitely stiff beams, although not discussed here[2]. The primary unknowns are the nodal displacements of the cantilever that, with ingenuity, enable a number of useful predictions to be made about the original structure. However, the more likely option is that a finite element analysis would be undertaken on the original frame. On the other hand, the arguments put forward when developing the model can be extended straightforwardly to cover the buckling and vibration of such frames. In this way, it is possible to use 'exact' member theory to determine both elastic critical buckling loads and the lower natural frequencies. In these cases, where the primary unknown is the most important parameter sought, the methods become very much more attractive. Furthermore, the arguments can be extended to cover three dimensional structures[3] that may additionally include braced frames, shear walls and cores. Such models are also in the process of being extended to allow for the possibility of doubly asymmetric floor plans[4]. Other models have also been proposed for static[5,6], buckling[7-11] and dynamic analysis[2,11-13].

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