Paper

Shear design of circular concrete sections using the Eurocode 2 truss model

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Synopsis

The introduction of the Eurocodes for concrete design will alter the way that shear is approached for concrete structures. BS EN 1992-1-1[1] has adopted the variable angle truss model for shear, a more theoretically consistent approach than that used in BS 8110-1[2]. The model is confidently applied to rectangular sections, but its applicability to irregular sections is less clear. In particular, the behaviour of circular concrete sections is not well defined.

This paper is intended to satisfy a requirement for design guidance on this topic that has been recognised by key BSI Committees. Using both experimental and theoretical data, the Eurocode variable angle truss model for shear design is assessed and extended to circular columns.

Symbols

A_{sv}	Cross sectional area of the legs of a link (BS 5400-4[3])
A_{sw}	Cross sectional area of shear reinforcement
B_w	Web width of equivalent rectangle for circular sections
D	Circular section outer diameter
$F_{b,c,i}$	Force in longitudinal bar i in compression
$F_{b,t,i}$	Force in longitudinal bar i in tension
F_c	Stress block compression force
F_{cd}	Force in the compression chord
F_{td}	Force in the tension chord
ΔF_{cd}	Additional tension in the compression chord
ΔF_{td}	Additional tension in the tension chord
I	Second moment of area (= $\pi D^4/64$ for circular sections)
M_{Ed}	Design value of the applied moment
N_{Ed}	Design value of the applied axial force
S	First moment of area above and about the centroidal axis
V_{Ed}	Design shear force
V_{Rd}	Design shear resistance
$V_{Rd,c}$	Design shear resistance of the member without shear reinforcement
$V_{Rd,max}$	Design value of the maximum shear force which can be sustained by the member, limited by crushing of the compression struts
$V_{Rd,s}$	Design value of the shear force which can be sustained by yielding the shear reinforcement
a	Shear span
b_w	Web width of the section
c	Depth of the compression chord
d	Effective depth
$f_{c,max}$	Maximum permissible compressive stress in concrete
f_{cd}	Design value of concrete compressive strength
f_{ctd}	Design value of concrete tensile strength
$f_{t,max}$	Maximum permissible tensile stress in concrete
f_{yv}	Characteristic strength of link reinforcement (BS 5400-4[3])
f_{ywd}	Design yield strength of the shear reinforcement
p	Spiral pitch
r	Radius to extreme fibre of circular section
r_s	Radius to centre of longitudinal bars
r_{sv}	Radius to shear steel
s	Stirrup spacing
s_v	Spacing of links along the member (analysis to BS 5400-4[3])
y_c	Distance from the neutral axis to the centroid of the compression chord
y_t	Distance from the neutral axis to the centre of the tension chord
z	Lever arm corresponding to the bending moment in the element under consideration
z_0	Distance from the centroid of the tensile forces to the centre of mass of the section
α	Angle between shear reinforcement and the beam axis
θ	Compression strut angle
σ_{cp}	Compressive stress from axial load
ω	Internal angle of circular segment

Introduction

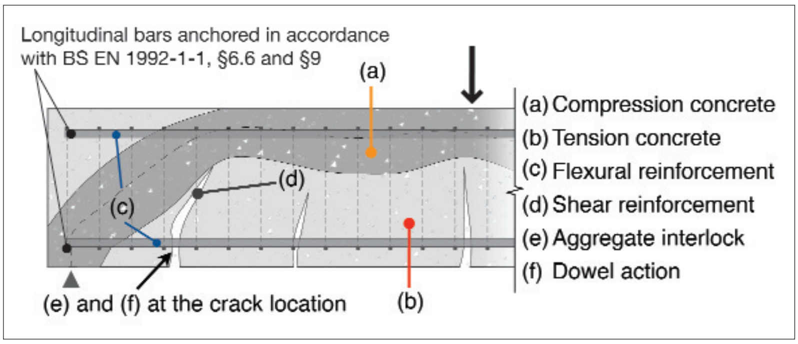
The circular concrete section is widely used in piling and bridge pier design, and its constant strength in all directions makes it useful in seismically active regions. Shear design in the UK has historically been based on Mörsch's 45°-truss model[4], while BS EN 1992-1-1[1] uses the more theoretically consistent variable angle model. The use, limitations, and application of this lower bound model to the design of circular sections in shear provides the basis for this paper.

Shear behaviour

Analysing shear behaviour is widely recognised as one of the more difficult aspects of reinforced concrete design. Shear failures are characterised by brittle action and are thus particularly critical when ductility at the ultimate limit state is a key design requirement. The design shear resistance of a concrete section can be considered as a synthesis of six contributing factors, as illustrated in Fig 1.

When present, shear reinforcement carries stress over cracks as they open under loading and confines the section. Aggregate interlock is estimated[5] to carry significant shear force (up to 50% of the capacity of the uncracked section), yet as cracks open the capacity to transfer stresses via aggregate interlock reduces. Finally, longitudinal reinforcement provides dowel action across shear cracks as they open.

The influence of axial load on shear capacity is complex, yet is critical for columns. Axial compression can be considered to flatten shear cracks, which then intersect more shear stirrups, increasing the section's shear capacity. Conversely, axial tension reduces the



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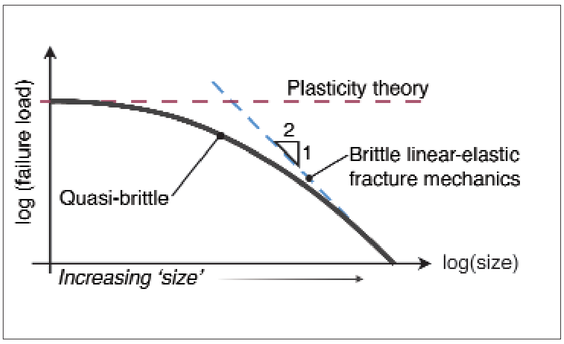

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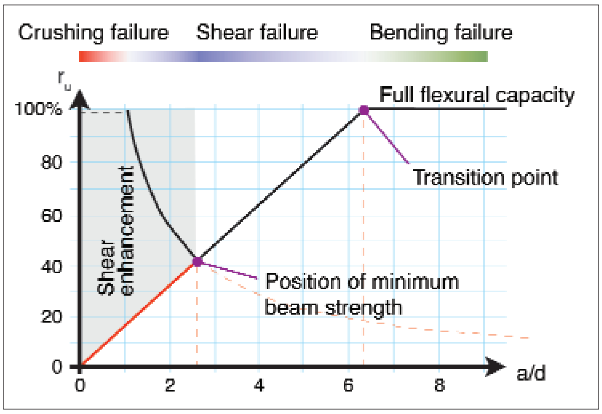

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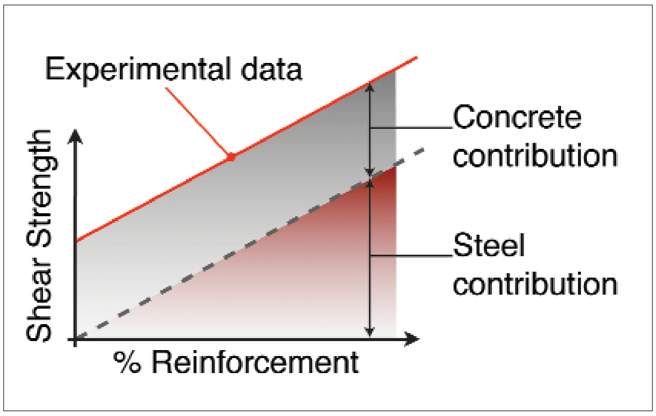

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1 Six contributing factors for shear capacity
2 The size effect
3 Kani's Shear Valley
4 Components of shear resistance in BS 8110-1

number of links intersected, and reduces the beneficial effects of aggregate interlock. However, it has been shown[6] that above a compressive stress of approximately 17MPa, the rate of increase in shear capacity due to axial compression reduces (and in some cases becomes negative) making linear relationships between shear capacity and axial load potentially non-conservative at high values of compressive stress.

A further important consideration for shear design is the size effect, first described by Kani[7], which highlights a reduction in strength of concrete members as they increase in size . Illustrated in Fig 2, it is best explained by considering that, regardless of member size, shear failures typically occur once shear cracks reach about 1mm in width, implying that larger elements are more brittle, albeit that the behaviour of an RC member is quasi-brittle, and does not follow a perfectly plastic – brittle linear-elastic path.

Shear failures typically occur over a relatively short region of the beam span, with a critical load position located at approximately 2.5*a/d*[7], where *a* is the shear span of the element under consideration. This area, sometimes referred to as the 'shear valley', is illustrated in Fig 3, where the relative beam strength (r_u, given by Kani[7] as the ultimate moment in the cross section at failure divided by the calculated flexural capacity of the cross section) is plotted against ratios of shear span to effective depth (*a/d*). A zone of shear enhancement occurs close to the supports ($a/d < 2.5$) whilst full flexural failure can only be achieved at higher values of *a/d*. In low span/depth ratio sections, the 'transition point' from shear to flexural failure may be unobtainable and thus the section is likely to fail in shear.

Truss analogy

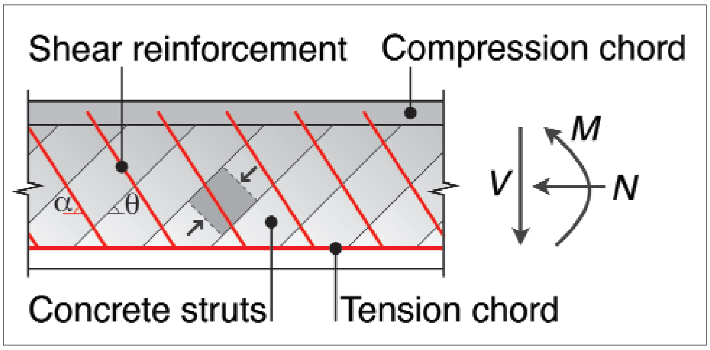
Modelling shear flow in a reinforced concrete section as a truss was first proposed by Ritter[8] and Mörsch[4] as a convenient design method, and is used in both BS 8110-1[2] and BS EN 1992-1-1[1]. The basic premise of the model is that cracked concrete in the web of the section resists shear by a diagonal uniaxial compressive stress in a concrete strut which pushes the flanges apart and causes tension in the stirrups which are then responsible for holding the section together. The Mörsch model assumes that the angle from the shear reinforcement to the beam axis is 90° and the angle of the compression strut from the beam axis is 45°. Early researchers[9] found that shear strength predictions according only to the force in the stirrups underestimated section shear strength by a fairly consistent value. To correct this, an empirically determined 'concrete contribution' to shear resistance was added, Fig 4, with this method being employed in BS 8110-1.

In the Eurocode model, a simplification is made such that once cracked, the section shear capacity comes solely from the links. However, the BS EN 1992-1-1 model allows the designer to select the angle of the compression strut (θ, Fig 5), with a flatter strut leading to more links being intersected, thereby increasing shear capacity. As θ decreases the compressive stress in the strut increases and the strut angle must therefore be limited to prevent concrete crushing, as illustrated by the Mohr's circle in Fig 6, where an increase in circle diameter corresponds to a decrease in the strut angle, θ.

Without a concrete contribution factor, the BS EN 1992-1-1 model can show appreciable step changes in capacity predictions as link spacing is increased, whereas in BS 8110-1 the concrete contribution tends to reduce the relative influence of the stirrups on overall shear capacity[10]. However, the variable angle model is generally considered to be a more consistent approach to shear design, as compression strut angles observed at failure in tests are generally flatter than 45°, as seen in the tests undertaken by Capon[11].

Alternative design methods

Recognising that the truss analogy is a simplified model of shear behaviour, new methods for shear design have been developed. Compression field theory (and its modified derivative, MCFT) is perhaps the most developed, originating from work by Wagner[12] on the post-buckling behaviour of metal beams with very thin webs where it was determined that post-buckling, the web of the metal beam no longer carries compression but instead resists shear by a field of diagonal tension. In concrete, the behaviour is reversed such that post-cracking the section no longer carries tension, instead resisting shear by a field of diagonal compression[13]. The method, which has been adopted in a simplified form by the Canadian Standards Association for the design of concrete structures and is detailed fully elsewhere[14], has been shown to be successful in the analysis of circular sections[10]. However, it is computationally more complex than the truss



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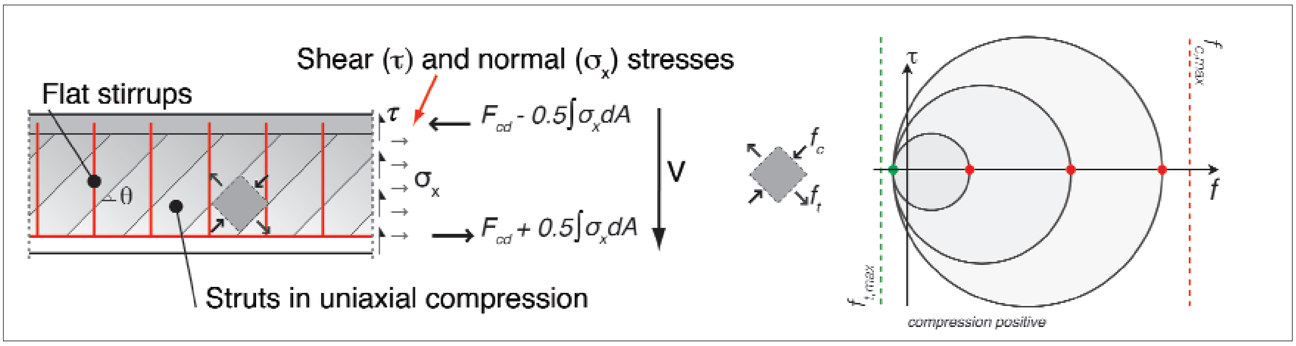
5 Simplified truss model
6 The effect of a variable strut angle



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analogy and, to date, has generally not been adopted by code writing committees.

Circular sections

Following the earthquakes of Mexico City (1957) and Coatzacoalcos-Jaltipan (1959), where a large number of circular columns were found to have failed in shear[15], Capon[11] undertook some of the earliest shear tests on circular concrete columns. However, of the 21 specimens tested, just four were transversely reinforced and only two of these failed in shear.

This work was later followed by Clarke[16] who undertook 97 separate tests on 50 circular specimens in shear to produce one of the largest available sets of experimental data for static loading. More recently, Jensen *et al*[17] tested 16 circular specimens, twelve of which were heavily reinforced with closed links (A_{sw}/s ranging from 1.01 to 4.52). There is thus a clear deficit of available test data for statically loaded shear reinforced circular sections and additional experimental research would be advantageous to further verify the proposed design method for circular sections that is described below.

Codified design

In BS EN 1992-1-1, shear reinforcement is required when the design shear force (V_{Ed}) is greater than the shear resistance of a section without stirrups ($V_{Rd,c}$). When $V_{Ed}>V_{Rd,c}$ the design shear resistance of the section is typically based on providing sufficient shear reinforcement ($V_{Rd,s}$), although the truss angle may be limited by crushing in the inclined concrete strut ($V_{Rd,max}$).

The truss analogy, as a lower bound approach, should satisfy equilibrium at all locations. Uniaxial compression in the inclined concrete strut must therefore be considered in terms of both its horizontal and vertical components, resulting in horizontal tension in the top and bottom chords of the truss model, in addition to the forces associated with flexure.

Such an approach requires that longitudinal reinforcement designed for flexure only (in accordance with BS EN 1992-1-1 §6.1) cannot also be used to resist the horizontal forces generated by the inclined compression struts. The Eurocode approach therefore differs from methods that consider the whole member as a truss for all loading cases, in which the forces in all the web and chord members may be determined by statics.

The design of shear reinforced circular sections can thus be considered to depend on 1) the capacity of the transverse steel, 2) the crushing capacity of the inclined struts ($V_{Rd,max}$) and 3) the additional tensile force in the distributed longitudinal steel (ΔF_{td}). These three criteria, which form the basis of the proposed design approach, are analysed in further detail below.

Flexural analysis

For rectangular sections without axial load, BS EN 1992-1-1 the assumption that $z = 0.9d$ may normally be made. However, this is not necessarily conservative for circular sections and a more accurate value for the internal lever arm may be determined by sectional analysis.

By first assuming a sensible neutral axis depth, the strain in the concrete at each layer of reinforcement is determined, from which the compression or tension forces in the reinforcement are obtained. Taking the BS EN 1992-1-1 stress block (Fig 7) the compression force in the concrete is given by Eq.(1) (note that the value of ηf_{cd} is reduced by 10% to satisfy BS EN 1992-1-1 cl.3.1.7(3)).

$$F_c = 0.9\eta f_{cd}\,[0.5r^2\,(\omega - \sin\,\omega)] \qquad \text{...(1)}$$

Where $(\omega - \sin\,\omega)$ is the area of the truss model compression chord, and, $\cos(\omega/2) = (r - \lambda x)/\,r$, Fig 7.

The compressive and tensile forces in the section are balanced to achieve equilibrium by iterating the neutral axis depth, x. The forces F_{cd} and F_{td} are located at distances y_c and y_t respectively from the neutral axis, with these values determined by moment equilibrium, Eq.(2) and Eq.(3). The internal lever arm, z, is then given by Eq.(4).

$$F_{cd}\,(y_c) = F_c\,y_{ci} + \Sigma F_{b,c,i}\,y_{ci} \qquad \text{...(2)}$$

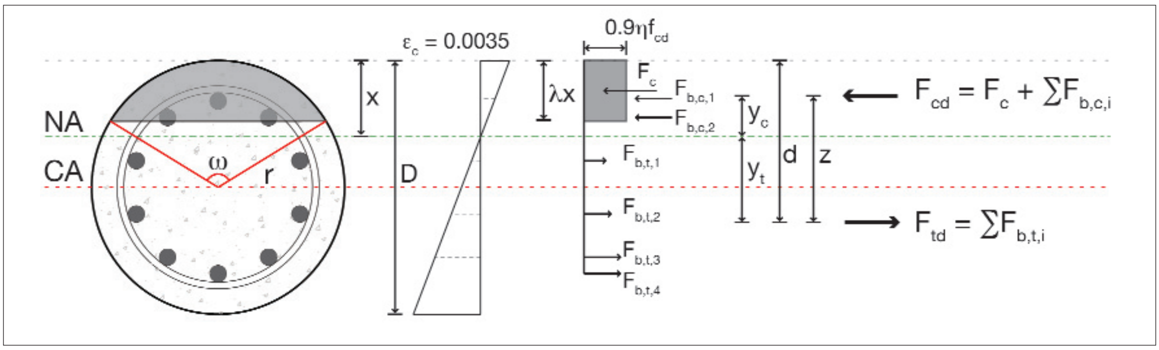
Where y_{ci} is the distance between the neutral axis and the compression force under consideration.

$$F_{ct}\,(y_c\,) = \Sigma F_{b,t,i}\,y_{ti} \qquad \text{...(3)}$$

Where y_{ti} is the distance between the neutral axis and the tension force under consideration.

$$z = y_t + y_c \qquad \text{...(4)}$$

The internal lever arm should be calculated for each section



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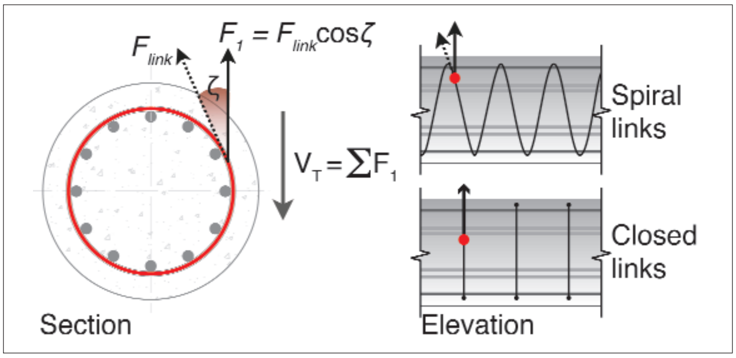

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7 **Flexural analysis to determine 'z'**
8 **Shear across a circular section**

under consideration. For a member loaded by a single point load, the critical section is likely to coincide with the position of maximum moment, while for a member under uniform load a section taken some distance from the supports may be more appropriate (cf. Fig 3 and Brown *et al*.[18]). Determining a value for z at the position of maximum moment and applying it along the entire length of the section is a conservative approach as this provides the largest compression chord area and hence the smallest value for z.

The sectional method described above thus ensures that the position of the truss model tension chord is determined based on all the steel below the section's neutral axis. This contrasts to previous approaches[11,16,19] in which the effective depth of the circular section was calculated by assuming the neutral axis to be coincident with the centroidal axis of the section.

By analysing 38 of the circular sections tested by Clarke[16], it was found that the assumption of $z = 0.9d$ (where d is the distance from the extreme compression fibre to the tension chord) consistently overestimates the lever arm of a circular section. An average value of $z = 0.77d$ (standard deviation, $\sigma = 0.021$, coefficient of variation, $c_v = 0.027$) was found using the method described above.

Transverse reinforcement

For geometric reasons, only a component of the force in a closed or helical link can resist applied shear forces, reducing their efficiency when compared to a rectangular link, as illustrated in Fig 8. Feltham[19] determined equations for both closed and spiral links, with the force component in spiral links also being resolved along the length of the member.

Priestley *et al*.[20] assumed the links to be effective over the full depth of the column, while Kowalsky[21] and Feltham[19] both assume that only a portion of the links are effective. Turmo *et al.*[22] considered a Eurocode based approach for both solid and hollow circular sections, obtaining more general results by allowing the stirrups to be effective over a variable depth. The proposed[22] efficiency factor for circular sections, λ_1, modifies the BS EN 1992-1-1 equation for $V_{Rd,s}$ as shown below:

$$V_{Rd,s} = \lambda_1 \frac{A_{sw}}{s} z f_{ywd} \cot\theta \qquad ...(5)$$

The value of λ_1, which is independent of the chosen truss angle (θ), is determined by numerical integration and depends only on the section geometry:

$$\lambda_1 = \int_0^1 \sqrt{1-((z_0 - zX)/r_{sv})^2}\, dX \qquad ...(6)$$

Making the assumption that the centroid of compression and centroid of tension in the section are equidistant from the centroidal axis, a further simplification is obtained by disregarding possible variations in effective depth along the length of the member[22]. Assuming a constant lever arm of $0.8D$ and taking r_{sv} as $0.45D$ an effectiveness factor of $\lambda_1 = 0.85$ was obtained[22], resulting in a BS EN 1992-1-1 equation for circular sections with closed links, Eq.(7):

$$v_{Rd,s} = 0.85 \frac{A_{sw}}{s} z f_{ywd} \cot\theta \qquad ...(7)$$

However, the assumption that the compression and tension forces are equidistant from the centroidal axis does not reflect the behaviour of the circular section as determined above and it is therefore recommended that λ_1 be determined using Eq.(6).

Spiral links

Turmo *et al.*[22] further analysed spirally reinforced circular sections, determining a second efficiency factor, λ_2, that is also applied to $V_{Rd,s}$:

$$V_{Rd,s} = \lambda_1 \lambda_2 \frac{A_{sw}}{s} z f_{ywd} \cot\theta \qquad ...(8)$$

Where λ_2 is given by:

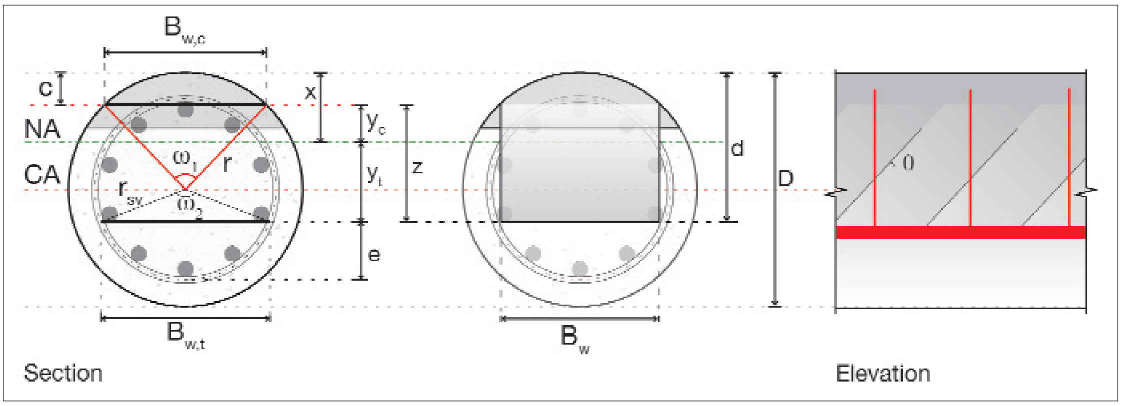
$$\lambda_2 = ((p/2\pi r_{sv})^2 + 1)^{-0.5} \qquad ...(9)$$

When the pitch, p, is set to zero (as for closed links), λ_2 is equal to unity and Eq.(8) is therefore applicable to both closed and spirally reinforced circular sections. Eq.(8) disregards the contribution to the shear capacity of the section by the longitudinal steel, which may be significant due to its distributed nature, and is therefore likely to represent a lower bound on the shear strength of a circular section.

Concrete crushing

The truss angle (θ) is limited in BS EN 1992-1-1 by crushing of the inclined concrete strut, Eq.(14), which is dependent on the state of stress in the compression chord (α_{cw}), the web width of the section

9 Crushing analysis geometry
10 ΔF in circular sections



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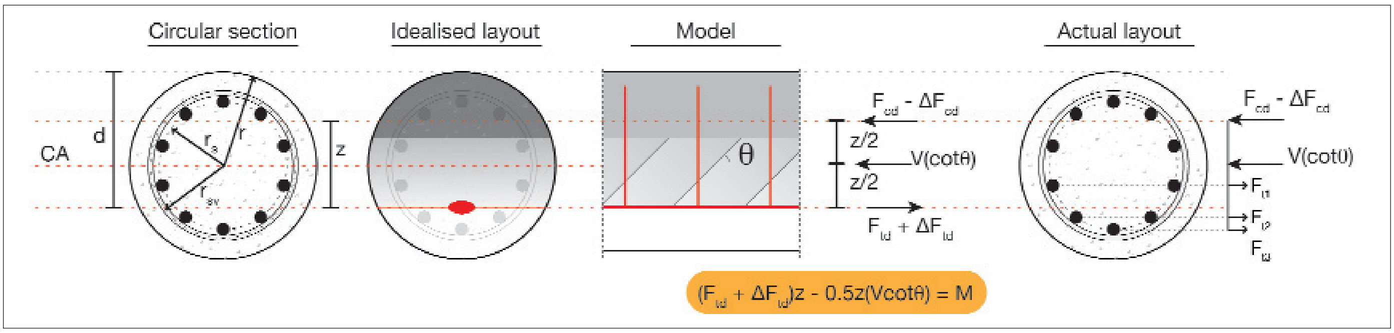

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(b_w), the lever arm between compression and tension zones (z) and the design strength of concrete that is cracked in shear ($\nu_1 f_{cd}$). Crushing in circular sections is considered here using an 'equivalent rectangle' approach to facilitate the use of the existing BS EN 1992-1-1 equations and a method to determine the equivalent web width for circular sections is presented.

The web width (B_w) of the equivalent rectangle is given by the minimum of the width of the section at the centroid of the compression chord, $B_{w,c}$ (Eq.(10)), and the width of the section inside the shear reinforcement at the centroid of tension, $B_{w,t}$ (Eq.(12)), both of which are illustrated in Fig 9.

$$B_{w,c} = 2\sqrt{c(2r - c)} \quad ...(10)$$

where $c = x - y_c = d - z$ ...(11)

$$B_{w,t} = 2\sqrt{e(2r_{sv} - e)} \quad ...(12)$$

where $e = \left(\frac{D + 2r_{sv}}{2}\right) - d$ ...(13)

The minimum of $B_{w,c}$ and $B_{w,t}$ is then used to analyse $V_{Rd,max}$ for circular sections, where the variables α_{cw}, ν_1, f_{cd} and cot(θ) are given by BS EN 1992-1-1 §6.2.3:

$$V_{Rd,\max} = \frac{\alpha_{cw} B_w z \nu_1 f_{cd}}{\cot\theta + \tan\theta} \quad ... (14)$$

where $B_w = \min\{B_{w,c}, B_{w,t}\}$

Additional tensile force, ΔF_{td}

The inclined concrete struts of the truss model are in uniaxial compression, resulting in a horizontal component of force which must be resisted by the longitudinal steel to satisfy equilibrium. In BS EN 1992-1-1, this force is applied at a position equidistant from the tension and compression chords of the truss model and each chord thus carries half the total horizontal force, equal to 0.5V cot(θ) (Fig 10).

The truss model chord forces should then be adjusted (Eq.(15)), with the result that the previously determined value for z should be modified to ensure equilibrium is satisfied.

$$F_{cd} = \frac{M}{z} - 0.5V\cot\theta \quad ...(15)$$

Theoretically, lower bound plasticity theory allows the additional tensile force to be distributed in any manner so long as the arrangement satisfies equilibrium at all points, balances the external loads and does not violate the yield condition (which defines the onset of plastic deformation). These conditions may be satisfied by increasing the force in all the longitudinal bars by a uniform amount in tension to account for shear. This method has the advantage that the resulting shift in neutral axis is likely to be small and could have a negligible effect on moment equilibrium.

Unreinforced section capacity

In sections where $V_{Ed} < V_{Rd,c}$, BS EN 1992-1-1 does not require the provision of design shear links. For rectangular sections that are cracked in bending, $V_{Rd,c}$ is determined by empirical formulae which have not been verified for use in the design of circular sections. For sections that are uncracked in bending the principal tensile stress in the section should be limited to the tensile strength of the concrete, Eq.(16).

$$V_{Rd,c} = \frac{lb_w}{S}\sqrt{(f_{ctd})^2 + \sigma_{cp} f_{ctd}} \quad ...(16)$$

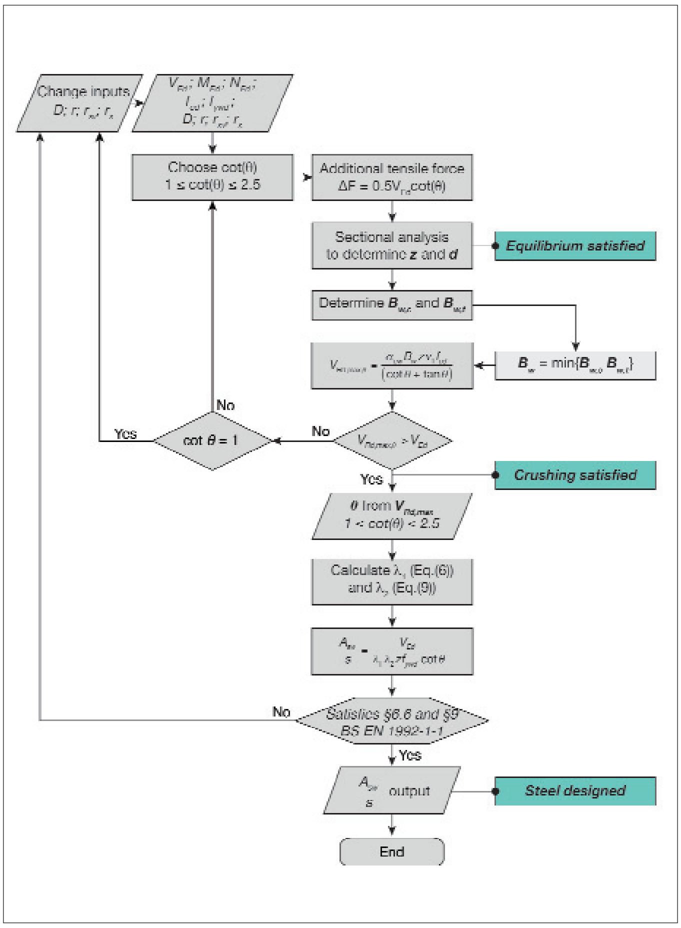
This approach is modified for use in circular sections as Eq.(17):

$$V_{Rd,c} = \frac{3\pi r^2}{4}\sqrt{(f_{ctd})^2 + \sigma_{cp} f_{ctd}} \quad ...(17)$$

For sections under high axial compressive stress, calculating $V_{Rd,c}$ using Eq.(17) is a sensible approach that is widely used in prestressed concrete design. Where applied axial loads are low, calculation of $V_{Rd,c}$ by this method will be conservative, since the contribution from the longitudinal steel is disregarded.

Design

The design process described by this paper for the design of shear reinforced circular sections using the variable angle truss model is presented in Fig 11, to be read in conjunction with Fig 12. The approach verifies that sufficient transverse links are provided, that the crushing limit is not exceeded, and that the additional tensile force is distributed according to the requirements of the



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factor. The transverse reinforcement yield stress was taken as f_{ywd} = 250MPa (J. Clarke, pers. comm., 9 March 2009). A ratio of theoretical to actual failure load of 3.32:1, with a standard deviation of 0.87, was found.

Clarke[16] presented test data for 17, 300mm diameter sections reinforced with 8mm diameter closed links at 150mm centres. Longitudinal reinforcement percentages ranged from 2.3% – 5.6%, while concrete strengths of between 24.10MPa and 48.40MPa were recorded. Shear capacity predictions using the proposed method range from 58.04kN to 58.80kN, while the recorded failure loads ranged from 145kN to 262kN[16].

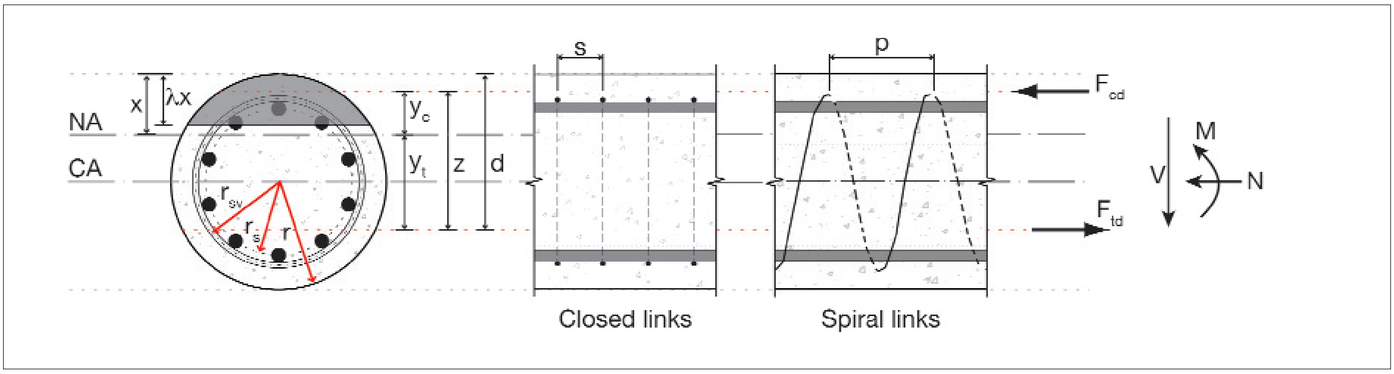
The results therefore show that whilst the proposed method is conservative, it is unable to account for variations in the concrete strength and percentage of longitudinal reinforcement in specimens with the same diameter and transverse reinforcement ratio.

For the tests undertaken by Clarke[16], the proposed design method is compared to an equivalent BS 5400-4 analysis in Fig 14, where the components of shear resistance arising from both the transverse reinforcement (V_s, Eq.(19)) and the concrete (V_c) (BS 5400-4, cl.5.3.3) are plotted.

$$V_s = \frac{A_{sv} f_{yv} d}{s_v} \qquad \text{...(19)}$$

Taking cot(θ) = 2.5, the Eurocode model would be expected to provide a shear capacity prediction approaching 2.5 times that predicted by the steel term (V_s) of BS 5400-4. However, it should

11 Design flowchart
12 Circular section dimensions, vertical links (left); spiral reinforcement (right)
13 Effectiveness of design equations for circular sections



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lower bound theorem.

In the approach summarised in Fig 11, shear reinforcement should be provided such that $V_{Rd,s} \geq V_{Ed}$. The calculation of $V_{Rd,s}$ for circular sections with closed or spiral links is undertaken by Eq.(18), where λ_1 is given by Eq.(6) and λ_2 by Eq.(9):

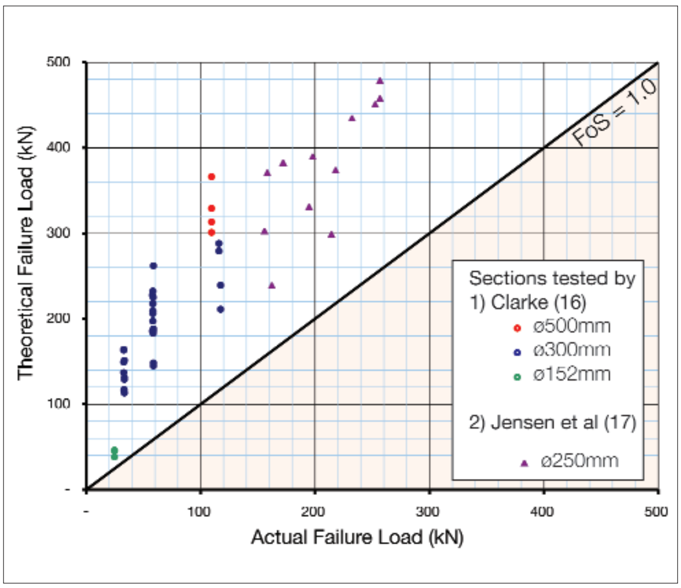
$$V_{Rd,s} = \lambda_1 \lambda_2 \frac{A_{sw}}{s} z f_{ywd} \cot\theta \qquad \text{...(18)}$$

The compression strut angle, θ, may initially be taken as 21.8° ($\cot\theta$ = 2.5). Should the section be found to fail by crushing of the inclined strut, θ may be increased within the limits of BS EN 1992-1-1 cl.6.2.3(2).

Application of the design method

The use Eq.(18) was compared to the experimental data provided by both Clarke[16] and Jensen *et al.*[17], with the results shown in Fig 13. The two analyses are discussed in the following.

38 columns tested by Clarke[16], ranging in diameter from 152-500mm that failed in shear and were reinforced with closed links, were assessed using the proposed design method. All partial safety factors were set to 1.00 and a strut angle of cot(θ) = 2.5 was taken in all cases as $V_{Rd,max}$ was not found to be the limiting



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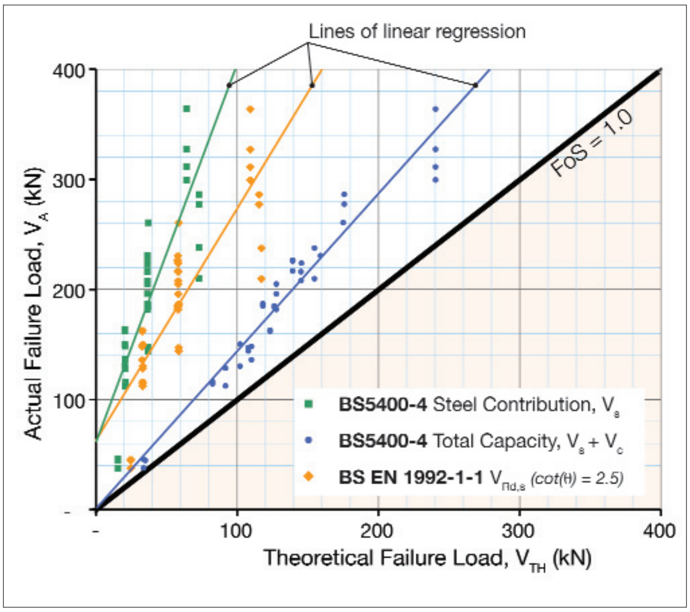

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14 BS5400-4 and proposed BS EN 1992-1-1 comparison

be noted that while Eq.(19) calculates V_s based on the effective depth of the section, Eq.(18) uses the smaller value of the lever arm between compression and tension chord forces.

Fig 14 shows that in an analysis to BS 5400-4[3] of the data presented by Clarke[16], V_s accounts on average for 30% of the total predicted shear capacity of sections with closed links that failed in shear (assuming f_y = 250MPa). The same analysis to BS EN 1992-1-1 (assuming cot(θ) = 2.5) may then be expected to provide a shear capacity prediction of approximately 0.3 x 2.5 = 75% of that predicted by BS 5400-4.

For the sections analysed it was found that the value of $V_{Rd,s}$ as determined in BS EN 1992-1-1 was on average 1.60 times the value of the V_s term given by BS 5400-4 (σ = 0.04, c_v = 0.025). This is somewhat less than might be expected, but is explained in the proposed model by considering that if $z = 0.77d$, $\lambda_1 \leq 0.85$ and cot(θ) = 2.5 for circular sections, then $V_{Rd,s}$ will be approximately 0.77 x 0.85 x 2.5 = 1.64 times the value determined for V_s in a BS 5400-4 analysis.

The BS EN 1992-1-1 equation for $V_{Rd,s}$ is linearly dependent on the yield strength of the transverse reinforcement used and thus greater accuracy may have been obtained if full steel coupon data was available for the transverse reinforcing steel used by Clarke[16]. This relationship is illustrated by considering the fictional analysis of two circular sections that vary only in the yield strength of their transverse reinforcement. Analysis to BS EN 1992-1-1, assuming cot(θ) = 2.5 and $V_{Rd,max}$ not to be a limiting factor, will predict a linear relationship between f_{ywd} and $V_{Rd,s}$. In the same analysis undertaken to BS 5400-4, an increase in f_{yv} results only in an increase in the relative contribution of V_s to the total shear capacity of the section, which remains partially dependent on the concrete contribution, V_c.

Although more limited in size, the tests undertaken by Jensen *et al*[17] provide a useful data set of circular sections with high transverse reinforcement ratios (A_{sw}/s ranging from 1.01 to 4.52). 12 tests were carried out on 250mm diameter, 1800mm long circular sections, all of which were transversely reinforced with closed links and were loaded in shear at 425mm from their supports. All sections had the same concrete strength (31.70MPa) and longitudinal reinforcement (eight 10mm diameter high yield deformed bars (f_y = 500MPa) and eight 20mm diameter DYWIDAG bars (f_y = 900MPa)). The recorded yield strength of the transverse reinforcement ranged from 573MPa to 584MPa.

The specimens tested by Jensen *et al*[17] were analysed using the method proposed in Fig 11. For 11 specimens, $V_{Rd,max}$ was found to be the limiting condition and the model truss angle was increased within the limits of BS EN 1992-1-1. The resulting shear capacity predictions are shown in Fig 13, and an average ratio of theoretical to actual failure load of 1.84:1 with a standard deviation of 0.27 was found. The results of this analysis suggest that the proposed method remains conservative in sections with higher transverse reinforcement ratios.

Conclusions

The introduction of the variable angle truss model in BS EN 1992-1-1 has changed the way that shear design is approached for concrete structures. This paper provides a logical extension to the Eurocode model for shear, which is confidently applied to rectangular sections, for the design of circular sections, as summarised in Fig 11. Methods for the analysis of 1) transverse steel capacity, 2) the additional tensile force in the longitudinal steel and 3) the limiting crushing capacity of the inclined compression struts are presented to satisfy equilibrium of the truss model.

It has been shown that whilst these methods provide a suitable lower bound analysis, care must be taken in any assumptions made. The 'equivalent rectangle' model for crushing in circular sections suggests that crushing failures are unlikely, although complete verification is not possible without test data. The proposed equation for $V_{Rd,s}$ is shown to provide reasonable accuracy, giving safe results when analysed against a wide range of experimental data.

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