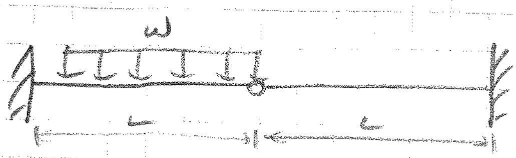
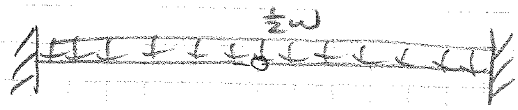
Deformation:

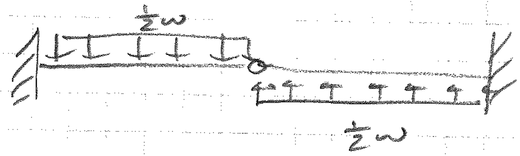
① 

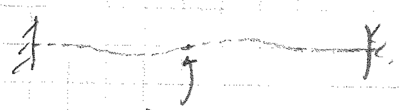
= =

② 

Symmetry 

\+ + +

③ 

Asymmetry. 

Note zero deformation at middle.

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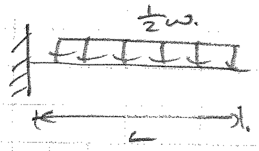
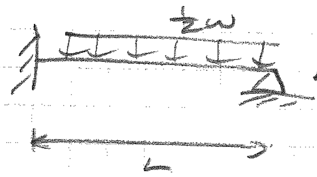
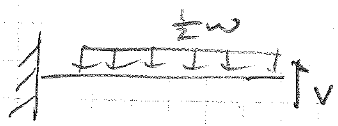
② can be represented as 

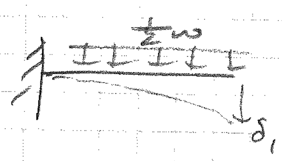
as (from symmetry) no shear exchange at pin.

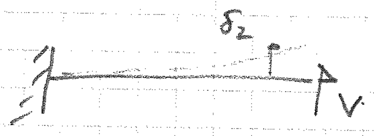
③ can be represented as 

← As no deformation due to asymmetry.

Indeterminate ⇒ to solve for shear exchanged at centreline pin need to consider stiffnesses.



= 

\+ 

If shear deformation neglected:

$$\delta_1 = \frac{\frac{1}{2}wL^4}{8EI} = \frac{wL^4}{16EI}$$

$$\delta_2 = \frac{VL^3}{3EI}$$

$\delta_1 = \delta_2$ as zero deflection at central support

$$\Rightarrow \frac{VL^3}{3EI} = \frac{wL^4}{16EI}$$

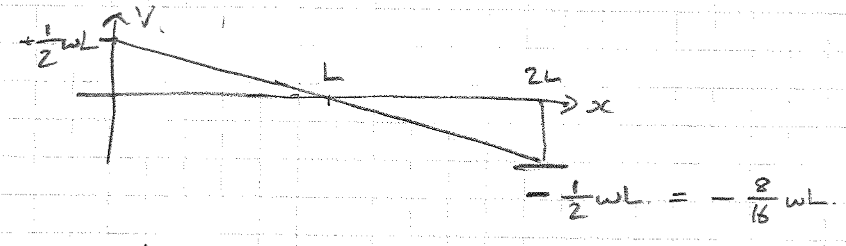
$$\frac{3}{16} = 0.1875$$

$$\Rightarrow V = \frac{3}{16}wL$$

ie approx 20% of load transferred to right-hand support.

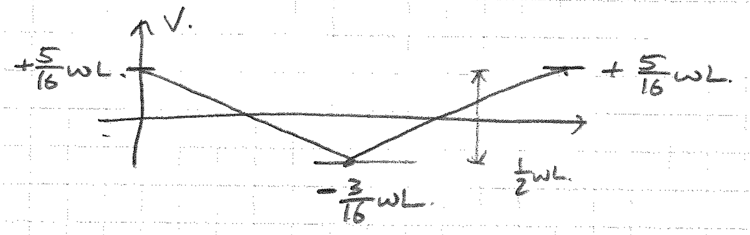
Shear force diagrams:

(2) :



+

(3) :



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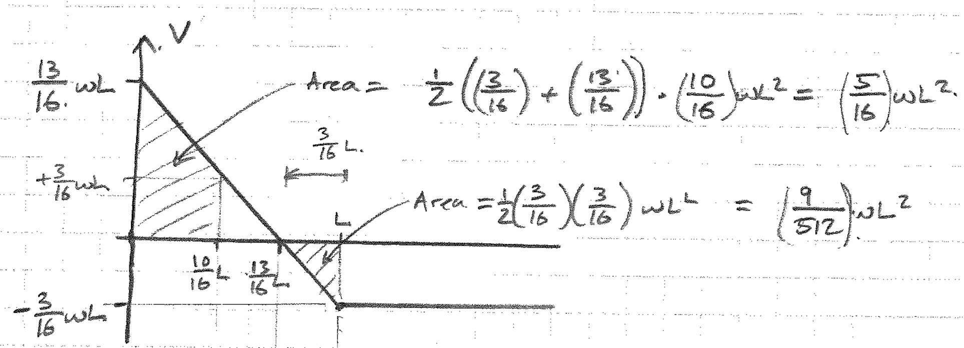
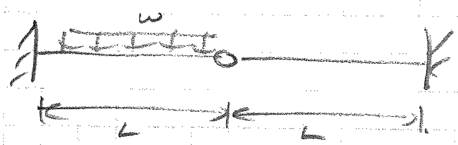
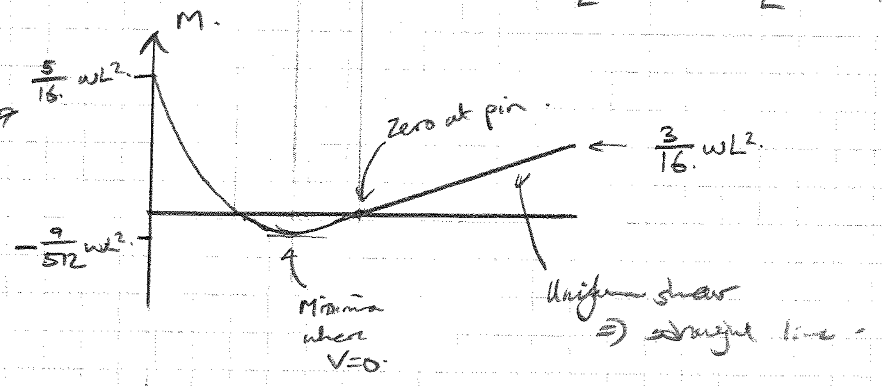
(1) :



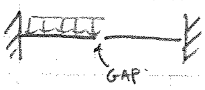
Moment diagram for (1), ie



situation:



c.f. $\frac{1}{2}wL^2$ if no connection between beams:



N.B:

$\frac{9}{512} \approx 0.018$

$\frac{3}{16} = 0.188$

$\frac{5}{16} = 0.313$

$\frac{1}{2} = 0.500$

<u>Extension</u>

Had shear deformation been included:

$$\delta_1 = \underbrace{\frac{\frac{1}{2}wL^4}{8EI}}_{\text{Flexure}} + \underbrace{\frac{(\frac{1}{4}wL)L}{GA_v}}_{\text{Shear}} = wL^2\left(\frac{1}{16}\left(\frac{L^2}{EI}\right) + \frac{1}{4}\left(\frac{1}{GA_v}\right)\right)$$

Average shear from integration of shear diagram.

$$\delta_2 = \frac{VL^3}{3EI} + \frac{VL}{GA_v} = VL\left(\frac{1}{3}\left(\frac{L^2}{EI}\right) + \left(\frac{1}{GA_v}\right)\right)$$

$\delta_1 = \delta_2$ as before.

$$\Rightarrow \quad VL\left[\frac{1}{3}\left(\frac{L^2}{EI}\right) + \left(\frac{1}{GA_v}\right)\right] = wL^2\left[\frac{1}{16}\left(\frac{L^2}{EI}\right) + \frac{1}{4}\left(\frac{1}{GA_v}\right)\right]$$

Multiply by $\frac{3EI}{L^2}$:

$$VL\left[1 + 3\left(\frac{EI}{GA_vL^2}\right)\right] = wL^2\left[\frac{3}{16} + \frac{3}{4}\left(\frac{EI}{GA_vL^2}\right)\right]$$

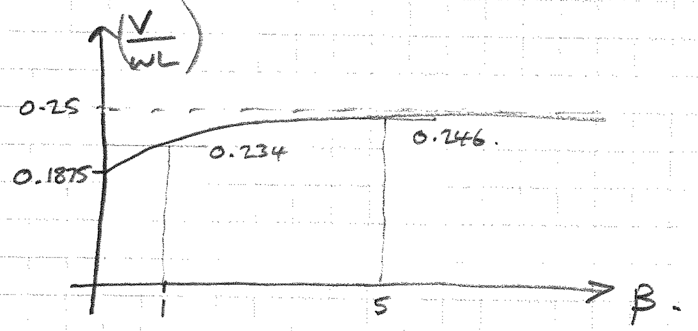
$$\Rightarrow \quad V = wL\left[\frac{\frac{3}{16} + \frac{3}{4}\left(\frac{EI}{GA_vL^2}\right)}{1 + 3\left(\frac{EI}{GA_vL^2}\right)}\right]$$

Define $\beta = \left(\frac{EI}{GA_vL^2}\right)$; dimensionless term relating bending stiffness to shear stiffness

$$\Rightarrow \quad \left(\frac{V}{wL}\right) = \left(\frac{3/16 + 3/4\,\beta}{1 + 3\beta}\right)$$

$\beta = 0$; $\left(\frac{V}{wL}\right) = \frac{3}{16}$

(This is solution when $(GA_v) \to \infty$, ie shear-stiff).



$\beta \to \infty$:

$$\lim_{\beta\to\infty}\left(\frac{V}{wL}\right) = \frac{\frac{3}{4}\beta}{3\beta} = \frac{1}{4}$$

(This is when $(GA_v) \to 0$, ie shear-flexible - eg. trusses)