

Size effect in frp reinforced concrete beams



Rory Ben O'Malley writes about his undergraduate project at Bath University that was Highly Commended in the 2004 Model Analysis Award

Reinforced concrete is a composite material. Compressive strength is provided by the concrete and is traditionally complemented by the tensile strength of inserted steel bars or tendons.

In most situations, the concrete provides a protective alkaline layer for the steel reinforcing bars. In aggressive environments such as marine structures and bridges treated with de-icing salts, this protection is lost, resulting in corrosion of reinforcing bars and sometimes a loss of serviceability. The cost of repairing steel reinforced structures damaged by corrosion in Europe has been estimated at £1bn/year¹.

Alternatives were sought to prevent the problem growing further; one of the most prevalent of the solutions proposed was the use of fibre reinforced polymers (FRPs) as an alternative reinforcing material. FRPs have the advantages of greater tensile strength, lightness, non-magnetic and, possibly most importantly, they are non-corrodible in normal service.

FRPs are not without disadvantages. FRP bars exhibit a relatively low stiffness when compared to steel, and more significantly are linear-elastic to failure. This means that current codified design guidelines for steel reinforced sections are largely inapplicable to FRP reinforced structures, because existing guidelines are based on the plastic behaviour of steel.

Size effect is a phenomenon concerning the ultimate shear stress of reinforced sections. As the size of a beam increases, its shear strength decreases. This effect is of particular concern when considering deep beams and very large structures.

The purpose of the research was to investigate the appearance, if any, of a size effect in concrete beams reinforced with FRP bars, and to test existing published guidelines that relate to the subject. Two existing FRP reinforcing guidelines are to be examined: the IStructE (1999) guide² and the ACI 440-01 (2001)³. Both propose that the

existing shear strength formulae can be modified for the use of FRP rebars.

The Institution of Structural Engineers published 'Interim guidance on the design of reinforced concrete structures using fibre composite reinforcement' in 1999². The guidance states that the area of longitudinal tension reinforcement, A_s , can be transformed into an effective area, $A_{e,s}$, for use in the existing BS 8110 shear strength expression. The transformation is calculated by multiplying by the modular ratio.

Concrete shear strength is obtained through the following equation from BS 8110⁴, with the substitution of A_e for the term A_s :

$$v_c = 0.79 \left(\frac{100A_e}{b_v d} \right)^{\frac{1}{3}} \left(\frac{f_{cu}}{25} \right)^{\frac{1}{3}} \left(\frac{400}{d} \right)^{\frac{1}{4}} / \gamma_n$$

f_{cu} should not be taken as more than 40N/mm² and the term $(400/d)^{1/4}$ should not be less than 1.0.

The term $(400/d)^{1/4}$ specifically determines effect of size on the shear strength, where an enhancement to the shear strength of a section is permitted for beams with effective depths of $d < 400$.

The American Concrete Institute committee 440 published the document 'Guide for the design and construction of concrete reinforced with FRPs bars' in 2001³. The publication states that the shear capacity of FRP reinforced beams can be predicted by modifying the existing ACI shear formula from ACI 318⁵, by using the following equation:

$$V_{c,f} = \left(\frac{\rho_f \times E_f}{90\beta_1 f'_c} \right) \times V_c$$

Where ρ_f = FRP reinforcement ratio, E_f = Young's modulus of FRP (psi), f'_c = specified compressive strength of concrete (psi).

The term β_1 is a modification factor of concrete strength of 0.85 for concrete strength up to 4000psi. For strengths above 4000psi, this factor is reduced continuously at a rate of 0.05 per 1000psi in excess of 4000psi, but is not

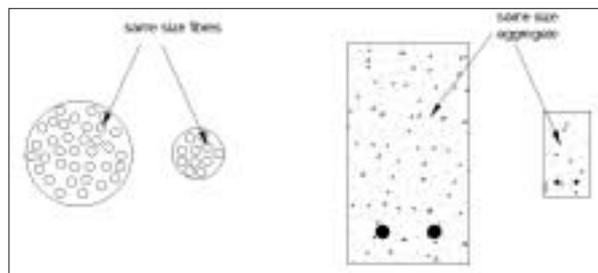
Table 1: Geometrically scaled beam series

Beam	Reinforcing bar diameter (mm)	Length (mm)	Depth (mm)	Breadth (mm)
A	10.5	3000	220	110
B	8	2280	168	84
C	6	1710	126	63
D	3.5	1000	73	37

Fig 1. (right) Beams B, C & D



Fig 2. (below) Scaling philosophy



to be taken as less than 0.65.

V_c is calculated from the existing ACI-318 'Building code requirements for structural concrete'⁵ using the following equation:

$$V_c = 2 \sqrt{f'_c} \times b_w d$$

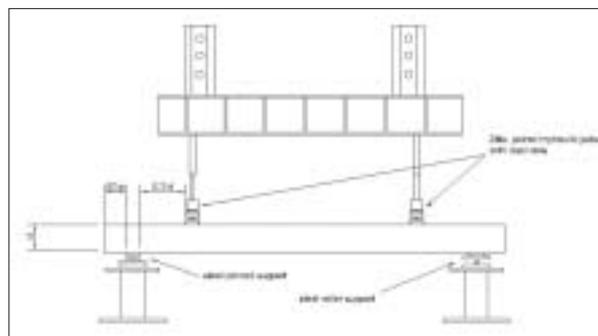
The terms in the formulae are either: constants, e.g., modulus of elasticity; or have a linear dependency on structure size, e.g., breadth or depth. This means when a series of shear strengths are calculated there will be no trend that displays a change with regard to increasing effective depth.

To test the effect of size, a beam series was required that was geometrically scaled (see Fig 1 and Table 1). The scaling of the beams was represented by their physical size, but not their constituent makeup (see Fig 2)

Table 2: Shear predictions from IStructE guidance² and ACI-440³

Beam	IStructE v_c (N/mm ²)	ACI-440 $v_{c,i}$ (N/mm ²)
A	0.703	0.151
B	0.749	0.155
C	0.808	0.155
D	0.924	0.155

Fig 3. General Loading arrangement



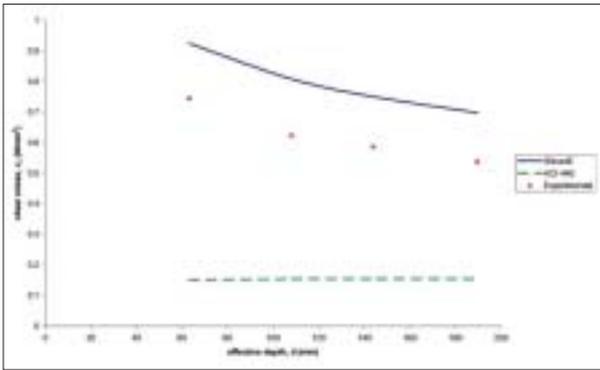


Fig 4. Predicted series shear stress against experimental



Fig 5. Catastrophic failure of Beam A



Fig 6. Dowel rupture of beam B (detail)

meaning the same concrete was used in all beams. A standard concrete mix was used (without plasticiser). The beams were cast on two separate occasions due to equipment constraints: the two mixes yielded concrete cube strengths on average of 62N/mm² and 59N/mm² respectively. Aramid fibre (AFRP) bars were used in this test series.

The beams were subjected to four-point loading as shown in Fig 3.

Shear predictions were obtained from the two design codes being tested, as shown in Table 2.

Test results summary

All tests were carried out as planned with no problems and failure modes were as expected. The initiation and propagation of flexural cracks was similar through the test series. Test results are summarised in Table 3.

There is clearly a size effect across the series, as shown by the decrease of shear strength compared to the effective depth of section in Fig 4.

Beams A and B suffered dowel rupture (see Figs 5 and 6), however

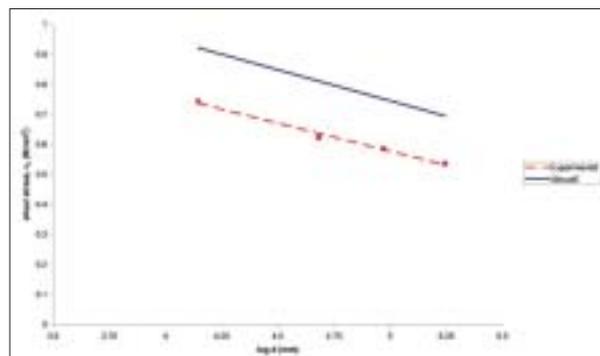
Beam	Average fcu (N/mm ²)	Failure mode	maximum deflection at mid-span (mm)	Shear force at failure	Shear strength (N/mm ²) (kN)
A	62	dowel rupture (RHS)	23.74	15.05	0.744
B	59	dowel rupture (RHS)	25.31	9.64	0.623
C	59	dowel-splitting of concrete (RHS)	26.26	5.77	0.586
D	59	dowel-splitting of concrete (LHS)	9.53	2.32	0.536

beams C and D experienced dowel splitting of the concrete. This may be because for the smaller beams, the relative tensile strength of the concrete must be lower compared to the size of beam, presumably because of the size of aggregate compared to the beam size. It is reasoned that a larger aggregate size generally would correspond to there being less adhesion between each discrete aggregate particle.

It can clearly be seen in Fig 4 that the IStructE recommendations predict trend of shear strengths accurately. Predictions were within a 4% band of variation, between 77-81% of experimental strengths. This suggests that, without safety factors present, IStructE guidance yields predictions approximately 20% below actual experimental results. Further testing would be required to confirm the absolute accuracy of the guideline however this result seems to show that with the inclusion of a safety factor of at least 1.2 would give extremely accurate predictions of shear strengths. This however is not best practice; for design purposes the recommended value for λ_{mv} is 1.25. This would give predictions ~5% above ultimate strengths, which seems low, however further testing would be required for clarification.

The IStructE guidance² provides an accurate estimation of the size effect of FRP reinforced concrete beams without stirrups. Size effect is modelled in BS 8110⁴ in the term $(400/d)^{1/4}$ of the concrete shear strength formula. This means shear strength has linear proportionality to effective depth when plotted on a logarithmic scale; the gradient of the line being -0.25. Fig 7 is a logarithmic plot of effective depth against shear strength. Whilst it is not strictly good practice to assume linear-

Fig 7. Shear strength versus log of effective depth



ity of a narrow band of experimental results, for the purposes of visualisation Fig 7 shows an idealised trend line.

It can clearly be seen here that the gradients of the plotted lines are nearly identical. Fig 7 shows that in this instance, the existing model for size effect in the British codes is an accurate model of the size effect in shear. The change of reinforcement has the same uniform decrease across the whole series, as modelled by the inclusion of the modular ratio factor. The size effect is specifically related to the fracture of concrete and is independent of the reinforcement type.

The ACI 440 formula³ does not model size effect in any way. The clear size effect present in actual results is not represented by the predictions, as shown in Fig 4. It also gives very conservative shear predictions, with values in the range of 18-25% of actual shear strength. This conservativeness is only compounded with the inclusion of proposed safety factors in calculation. This will lead to over designed structures and inefficient material use.

Shear strength of a reinforced concrete beam is strictly related to reinforcing material however the size effect that is inherent in shear is independent of the material used: FRP reinforced sections exhibit a pronounced size effect in shear capacity, very similar to that displayed in steel reinforced sections.

REFERENCES

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