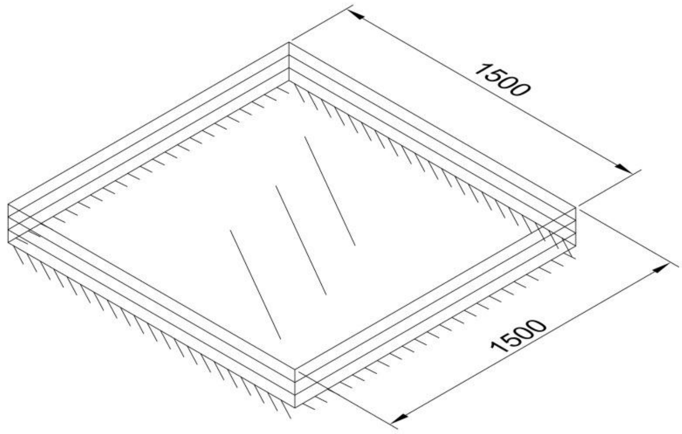
Worked example for floor plate design

A glass floor plate is to be installed that sits over a basement. The support is via a steel frame with a 1.5m x 1.5m grillage of beams. This frame provides a continuous support and the following analysis and design of the glass floor plate provisionally assumes it to be infinitely stiff. The following calculation determines the thickness of the glass required for the floor plate.



The below floor loadings are typical for domestic use and should be used for design:

Characteristic variable action $q_k = 1.5kN/m^2$
Characteristic point action $Q_k = 3.0kN$

The glass will not be sandblasted and will be enamelled on the wearing side. This will not impact on the design of the glass as the enamelling is on the compression side of the floor plate.

The glass is to be made up of three layers of glass laminated with PVB, It is assumed that the top sheet will be toughened and not included in the stress calculations although will be considered for deflection. To provide the most effective post breakage behaviour the lower two sheets will be heat strengthened.

Two action durations apply: permanent and short.

Permanent action condition

Determine design strength of glass using Appendix C.

$$f_{g;d} = \frac{k_{mod}k_{sp}f_{g;k}}{\gamma_{M:A}} + \frac{k_v(f_{b;k} - f_{g;k})}{\gamma_{M:v}}$$

For load duration > 50 years, $k_{mod} = 0.29$

$k_{mod} = 0.29$, $k_{sp} = 1.0$, $f_{g;k}$=45 N/mm² $f_{b;k} = 70N/mm^2$, $k_v = 1.0$, $\gamma_{M:A} = 1.6$ and $\gamma_{M:v} = 1.2$.

$$f_{g;d} = \frac{0.29 \times 1.0 \times 45N/mm^2}{1.6} + \frac{1.0(70N/mm^2 - 45N/mm^2)}{1.2} = 29N/mm^2$$

Variable action condition i.e. short duration load

For load duration of 5 hours, for pedestrian action; $k_{mod} = 0.60$

$$\therefore f_{g;d} = \frac{0.60 \times 1.0 \times 45N/mm^2}{1.6} + \frac{1.0(70N/mm^2 - 45N/mm^2)}{1.2} = 37.7N/mm^2$$

Determine the effective thickness of the glass required based on design strength

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Try two 12mm thick plies and one 12mm thick sacrificial ply. The laminate is a PVB based material and is 0.76mm thick.

In order to calculate the effective thickness of the glass pane, the shear interaction between the plies of the laminated glass via the PVB interlayer must be accounted for.

The simplified method of determining stress of glass within each ply is as follows.

The effective thickness of the glass when considering deflection due to bending is:

$$h_{ef;w} = \sqrt[3]{\sum_k h_k^3 + 12\omega\left(\sum_i h_k h_{m,k}^2\right)}$$

where:

h_k is the thickness of each ply

$h_{m,k}$ is the distance between the middle of the ply to the centre of the laminated glass pane.

When calculating stress the equivalent thickness is calculated from:

$$h_{ef;\sigma;j} = \sqrt{\frac{h_{ef;w}^3}{h_j + 2\omega h_{m;j}}}$$

where:

h_j is the thickness of each ply

$h_{m,j}$ is the distance between the middle of the ply to the centre of the laminated glass pane.

Assuming a value of 0.1 for ω when considering variable actions and '0' when assessing effects due to permanent actions, two effective thicknesses need to be calculated.

Effective thickness in permanent action condition

The effective thickness of the laminated glass with respect to permanent action is as follows:

$$h_{ef;w} = \sqrt[3]{12^3\text{mm} + 12^3\text{mm}} = 15.1\text{mm}$$

Note that the variable ω equates to '0' hence it is not included in the calculation. Additionally the toughened glass top ply is not included when determining the overall depth of the glass as it is considered to be sacrificial.

Effective thickness in Variable Action Condition

The effective thickness of the laminated glass with respect to variable action is as follows:

$$h_{ef;w} = \sqrt[3]{\begin{array}{c}12^3\text{mm} + 12^3\text{mm} + 12 \times 0.1 \times (12\text{mm} \times \\ 6.4^2\text{mm} + 12\text{mm} \times 6.4^2\text{mm})\end{array}} = 16.7\text{mm}$$

This is the effective thickness of the laminated glass that is to be used for deflection calculations.

The effective thickness of the laminated glass for bending stress is as follows:

$$h_{ef;\sigma;j} = \sqrt{\frac{16.7^3\text{mm}}{12\text{mm} + 2 \times 0.1 \times 6.4\text{mm}}} = 18.7\text{mm}$$

Note how the thickness has increased from 15.1mm to 16.7mm due to the inclusion of shear interaction within the PVB based interlayer, which is then increased to 18.7mm when calculating stress

Bending stress check

There are two action conditions to check for: permanent and variable (short). With respect to the permanent condition, the following applies:

Permanent Actions due to self-weight = 0.036m x $25kN/m^3$ = $0.9kN/m^2$
(**Note** this includes the 12mm thick toughened glass top ply which is not included when determining the effective thickness of the glass.)

Partial factor for permanent action γ_q=1.35

$UDL_{ult} = 0.9kN/m^2 \times \gamma_g = 0.9kN/m^2 \times 1.35 = 1.22kN/m^2$

The applied bending stress to the plate during the permanent condition is defined in Table 11.4 of Roark's Formulas for Stress and Strain – 8th Edition as:

$$\sigma_{max} = \frac{\beta q b^2}{{h_{ef;w}}^2},\ \beta = 0.287$$

The β variable is dependent upon the ratio of geometry of the plate. In this instance the ratio of the dimensions of the plate is 1; therefore β = 0.287

$$\therefore \sigma_{max} = \frac{0.287 \times 1.22kN/m^2 \times 1.5^2m}{0.0187^2m} = 2.3\times10^3kN/m^2 = 2.3N/mm^2$$

$2.3N/mm^2 < 29N/mm^2$ ∴ OK

The applied bending stress to the plate during the variable action condition is:

$1.22kN/m^2 + 1.5\ kN/m^2 \times \gamma_q = 1.22kN/m^2 + 1.5\ kN/m^2 \times 1.5 = 3.47kN/m^2$

There are two effects to consider for the variable action condition: that of the UDL and the point load.

UDL

$$\sigma_{max} = \frac{0.287 \times 3.47kN/m^2 \times 1.5^2m}{0.0187^2m} = 6.4\times10^3kN/m^2 = 6.4N/mm^2$$

$6.4N/mm^2 < 37.7N/mm^2$ ∴ OK

Concentrated point action

The area over which the action is applied is 50mm x 50mm = $2500mm^2$

∴ equivalent circle area radius $r_0 = r'_0$ if $r_0 \geq 0.5t\ \therefore r'_0 = \sqrt{\frac{2500mm^2}{\pi}}$ = 28.2mm

Bending stress at point of concentrated load:

$$\sigma = \frac{3W}{2\pi {h_{ef;\sigma;j}}^2}\left((1+\nu)\ln\frac{2b}{\pi r'_0} + \beta\right)$$

Where:

W is the design concentrated point action
ν is Poisson's Ratio

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β is a variable that is dependent upon the ratio of geometry of the plate. See Table 11.4 of Roark's Formulas for Stress and Strain – 8th Edition.

In this instance the ratio of the dimensions of the plate equate to 1, therefore β=0.435

Partial factor for variable concentrated action γ_Q=1.5

$$\therefore \sigma_{point\ load} = \frac{3 \times 3000N \times 1.5}{2 \times \pi \times 18.7^2 mm}\left[(1 + 0.22)\ln\frac{2 \times 1500mm}{\pi \times 28mm} + 0.435\right] = 29.1N/mm^2$$

∴ including self-weight σ_{oa} =29.1N/mm^2 + 2.3N/mm^2=31.4 N/mm^2

31.4N/mm^2 < 37.7N/mm^2 ∴ OK

Deflection check

It Is safe to assume all three panels of glass participate in restricting deflection
Equivalent thickness of 3 layers is as follows:

$$h_{ef;w} = \sqrt[3]{12^3mm + 12^3mm + 12^3mm + 12 \times 0.1 \times (12mm \times 12.8^2mm + 12mm \times 0^2mm + 12mm \times 12.8^2mm)} = 21.5mm$$

Equivalent I value = $\frac{1500m \times 21.5^3mm}{12} = 1.24 \times 10^6mm^4$

Taking the point load as the worst case condition:

Serviceability state deflection =

$$\frac{3000N \times 1500^3mm}{48 \times 70000N/mm^2 \times 1.24 \times 10^6mm^4} = 2.4mm$$

Assuming a span/depth ratio of 250, the maximum allowable deflection is 6mm > 2.4mm therefore the floor plate passes both serviceability and strength checks.

Post-failure check

As all three sheets of glass could fail the only truly reliable test for the proposed design is to undergo physical testing of the design to confirm its suitability.

Some schools of thought suggest a design method where one sheet is assumed to remain intact. This can offer some measure of comfort but does not cover the case where all three sheets are broken. This approach is illustrated below.

Two action durations apply: permanent and short.

Permanent action condition

The strength of the glass is the same for the intact floor plate.

Bending stress check

In the post-failure condition there remain the two action conditions to check for: permanent and variable (short). With respect to the permanent condition, the following applies:

Actions are similar to intact condition i.e. 0.9 kN/m^2

Partial factor for permanent action γ_g=1.0 due to accidental condition

$UDL_{ult} = 0.9kN/m^2 \times \gamma_g = 0.9kN/m^2 \times 1.00 = 0.9kN/m^2$

When taking into account the two way spanning action of the floor plate, the applied bending stress to the plate during the permanent condition is:

$$\sigma_{max} = \frac{\beta q b^2}{h^2_{ef:w}}, \ \beta = 0.287$$

$$\therefore \sigma_{max} = \frac{0.287 \times 0.9kN/m^2 \times 1.5^2 m}{0.012^2 m} = 4.0 \times 10^3 kN/m^2 = 4.0N/mm^2$$

$4.0N/mm^2 < 29.0N/mm^2 \therefore$ OK

The applied bending stress to the plate during the variable action, short duration condition is:

Partial factor for varible action γ_q=1.0 due to accidental condition

$UDL_{ult} = 0.9kN/m^2 + 1.5kN/m^2 \times \gamma_q = 0.9kN/m^2 + 1.5kN/m^2 \times 1.0 = 2.4kN/m^2$

There are two effects to consider for the variable action condition: that of the UDL and point load.

UDL

$$\sigma_{max} = \frac{0.287 \times 2.4kN/m^2 \times 1.5^2 m}{0.012^2 m} = 10.8 \times 10^3 kN/m^2 = 10.8N/mm^2$$

$\therefore$ including self-weight $\sigma_{oa} = 10.8N/mm^2 + 2.3N/mm^2 = 13.1 \ N/mm^2$

$13.1N/mm^2 < 37.7N/mm^2 \therefore$ OK

Point load check

Although this is an accidental condition, the point load check still applies.

Partial factor for variable concentrated action γ_Q=1.0

$$\sigma_{concentrated\ action} = \frac{3 \times 3000N \times 1.0}{2 \times \pi \times 12^2 mm}\left[(1 + 0.22)\ln\frac{2 \times 1500mm}{\pi \times 28.2mm} + 0.435\right] = 47.1N/mm^2$$

$\therefore$ including self-weight $\sigma_{oa} = 47.1N/mm^2 + 2.3N/mm^2 = 49.4 \ N/mm^2$

$49.4N/mm^2 > 37.7N/mm^2 \therefore$ fails

If the two bottom sheets remain as 14 mm and the top sheet is dropped to 8mm then the overall thickness will remain the same and the stress calculated above will reduce to 36.9N/mm^2, which is within the allowable design bending stress.

No deflection check required as it is a post-failure condition.

The overall thickness of the glass is 37.5mm, which correlates closely to the span/depth ratio of 40.