Structural Vulnerability and Robustness[†]

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ABSTRACT: The ability of a structure to resist disproportionate collapse is an aspect of robustness. Existing structures which were either built before the current code provisions or have deteriorated may not be acceptably robust. An insight into the lack of robustness may be gained through a theory of structural vulnerability. The theory is based on an analysis of the form of a structure. Such an analysis leads to an identification of vulnerable failure scenarios which can be examined for the associated risks due to all possible actions. Appropriate action plans can then be developed to reduce the risks to an acceptable level.

1 INTRODUCTION

The aim of a design process is to achieve an acceptable probability that structures being designed will perform satisfactorily during their intended life. Accordingly, they are designed to sustain all the loads and deformations of normal construction and use. A further requirement of the current codes of practice (e.g. BS8110-1:1997, BS5950-1:2000) is to ensure the robustness of the structures so that damage to small areas of a structure or failure of single elements would not lead to the collapse of a major part of the structure. This is achieved through the provision of horizontal and vertical ties and, if necessary, through the identification of 'key elements' and designing them appropriately. However, the identification of key elements is not a trivial exercise. Existing structures which were either built before the current code provisions or which have deteriorated, pose interesting challenges. A recent report by IStructE (2002) has also recommended that better understanding of the vulnerability of building structures to progressive collapse is required.

The objective in this paper is to introduce an approach for the protection of structures against unforeseen events. First, a theory of vulnerability of structures, previously reported (Wu *et al* 1993, Lu *et al* 1999, Agarwal *et al* 2001) is introduced. It forms a basis for finding failure scenarios that are later examined for the chance of failure under specific actions and loads. Second, different actions - natural or man-made, are considered that have the potential to cause any of the failure scenarios identified. Third, a risk assessment is carried out considering the consequences of different vulnerable scenarios and the probabilities of actions causing them. This information is used to suggest an appropriate action to reduce the risk to an acceptable level. An example is given to illustrate the proposed approach.

2 STRUCTURAL VULNERABILITY THEORY

Progressive collapse is a property not only of the response of the structure to loading but it also depends upon the physical form of the structure i.e. the members and the way they are connected together. Every engineer knows that a statically indeterminate structure is more robust than a statically determinate structure. The degree of redundancy, stiffness of members, type of joints etc. are all related to the form of a structure. An inappropriate choice of the structural form can lead to a structure vulnerable to progressive collapse.

Structural vulnerability theory is a theory of form. It analyses the way a structure is connected together and identifies links which if damaged may lead to a progressive failure. In brief, it requires the following processes:

(i) Building a model of the form of a structural system: Graph theory is used as a basis to develop a model of the form of a structural system. The characteristics of the form of a structural system are captured in a structural ring in 2D or a structural round in 3D. This enables the development of a measure of the quality of its form.

(ii) Representing the structure in a hierarchical form: According to the quality of the form of the structural components, they are organized into structural clusters. Thus a structural system is represented as a set of interconnected clusters. Successive clustering results in a hierarchical form of the structure. At the lowest level a structure is represented with its components and at the highest level is the whole structural system.

(iii) Identifying various failure scenarios of the structural system: A hierarchical model reveals the form of the structure and enables the identification of weak links at different levels. A set of such links gives a failure scenario. By analyzing the damage

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demand and the consequences of various damage events, vulnerable failure scenarios are identified.

Key concepts to the vulnerability theory are (a) rings and rounds, (b) well-formedness, (c) clusters and hierarchy and (d) failure scenarios.

2.1 *Rings and rounds*

The function of a structure is to transmit force from one point to another in space without losing its integrity. Form is essential to that integrity i.e. a structure is able to maintain its load paths. A structural ring or a structural round is a minimum structural path which can withstand a set of forces in 2D and 3D respectively. Each of these has sufficient degrees of freedom to maintain equilibrium. For a pin-jointed structure, a ring has three members and this ring in itself is just one load path. In a graph model, a structural ring is a pattern which defines a valid structure (a load path). A structural ring is in a state of failure if it cannot carry any forces i.e. when it becomes a mechanism. The state of a structural ring can be changed by releasing one degree-of-freedom (DOF). As each DOF is released, there is a process of damage leading to failure. Each step is called a deterioration event. An over-stiff ring deteriorates into a just-stiff ring when all the redundant DOFs are lost. A just-stiff ring fails with the loss of a one more DOF. A typical structure will usually have several structural rings at different levels in the hierarchical model. Thus total failure of the structure may require several deterioration events. Depending upon where a deterioration event takes place, a structure can fail in many different ways. The deterioration hierarchy of structural rings includes all possible patterns in which an over-stiff ring deteriorates into a mechanism.

2.2 *Well formedness*

A structural ring or a round is only a pattern in the graph model of a structure. The quality of its form is a function of the member properties, their orientation and connectivity. This quality is called well-formedness and it is obtained using the stiffness sub-matrices associated with the joints in a structural ring. Wellformedness (q_i) of an *i*th node with stiffness sub-matrix $\mathbf{K}_{ii}$ is obtained as

$$q_i = \det(\mathbf{K}_{ii}) \tag{1}$$

This measure, though approximate for simplicity, has all the requisite features and it has been found to be satisfactory for a large class of 2D and 3D problems examined so far. This measure is closely related to the principal stiffness directions indicating the ability of a structural ring to resist loads from an arbitrary direction. Figure 1 illustrates how the well-formedness of an existing structure can decrease. Any change in wellformedness of an existing structure over a period of time would indicate damage to the structure. Of course, this requires a condition survey of the structure so that member properties and their connections can be represented in the structural model.

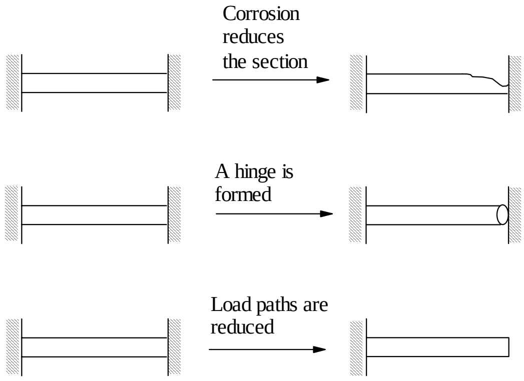

Figure 1. Factors affecting well formedness

2.3 *Clustering*

Clustering starts from the elementary level and fuses members (an object at the lowest level) to form clusters (an object at a higher level of description). When a cluster at a higher level of description is created, it captures certain attributes of those objects which formed it and becomes an individual object at that level. All objects at the elementary level finally evolve, through different clusters at a finite number of levels of description, into a single cluster which represents the whole structure. Such a hierarchical representation results in continuing selective loss of detail up the hierarchy. It is a map showing how the clusters inter-relate. The well formedness (Q) of a cluster, with N nodes, is obtained as

$$Q = \frac{1}{N}\sum_{i=1}^{N} q_i \tag{2}$$

The attributes of a structural member (or cluster) used for cluster formation include well formedness of the structural cluster, minimum damage demand of the structural cluster, nodal connectivity of the structural cluster and distance from the reference of the structural cluster. A reference cluster is not only a part of a structural system but also a reference point from which the scale of damage is measured. Some clusters which are further away from reference can be damaged through damage to clusters which connect them to the reference cluster.

2.4 *Damage*

A deteriorating event has an associated concept of damage demand (D) which measures the effort required to achieve that event. Based on form of a structure, this is a function of the principal stiffness coefficient corresponding to the degree of freedom which is lost as a result of the deteriorating event. The detail and nature of the actions causing that event are not considered. These may arise, for example, due to excessive loading, natural disaster, accidental force or human errors. Deteriorating events affect structural form and they can cause failure at the most elementary level. When such failure occurs, it can cause the complete breakdown of the structural system. The consequences of failure of different clusters vary and they largely depend upon how that cluster is configured in the system. The references (i.e. supports) play an important role in determining the failure consequence. For the same structure, the position of reference cluster can change the consequences drastically. The description of damage scale is related to the change in quality of form which is quite appropriate as it is a theory of form. Consequence (C) i.e. the loss in structural well formedness, is calculated as the difference between the structural well formedness of intact structure and the damaged structure.

2.5 *Failure scenarios*

A failure scenario is a set of deteriorated events ending with a mechanism. Five different failure scenarios are identified. Minimal failure scenario is composed of the least well-formed cluster at a level of definition. The minimum demand scenario gives the easiest possible way to cause damage to a structure at that level irrespective of its consequences. The latter aims to locate the weakest link at that level of definition. In a total failure scenario the structure fails completely. Maximal failure scenario is a scenario with the maximum consequence with the least damage. The ratio of consequence and damage demand is referred to as vulnerability index (V.I.). A maximal failure scenario need not be total. Other scenarios may include those for specific damage cases. Based on these concepts a computer program has been developed for the vulnerability analysis of a structure.

3 ACTIONS AND THEIR EFFECTS

There may exist different events or combination of events that may lead to catastrophic consequences in a vulnerable structure. These may include (a) degradation of material e.g. corrosion of steel, (b) extreme natural hazards such as hurricanes, (c) vehicle impact, (d) intentional damage e.g. terrorist bombing, (e) fire and many more.

In vulnerability theory, the measure of damage relates to the properties of a member. Actions causing damage are not specified and damage may result from any possible action. These may be naturally occurring actions or man-made actions. Of course, some actions have the potential to cause damage to many structural members simultaneously. It is possible, at least qualitatively, to assess the likelihood of certain actions.

Location of members is an important aspect affecting the likelihood. For example, columns close to access roads have higher possibility of vehicular impact or intentional damage. Exterior members are more prone to weathering effects such as corrosion. Similarly, interior members in a building may be more exposed to certain undesirable actions depending upon the usage of an area.

Proximity of stresses under normal design to the permissible stresses is another important factor. If the extreme values of commonly considered actions e.g. wind or earthquake, were to occur then highly stressed members are more prone to damage.

Although damage demand of a member is included in the vulnerability analysis, there remain certain aspects of damage which are difficult to quantify, for example, lateral restraints affecting buckling behaviour. Loss of lateral restraints is an action in this context and its likelihood can be estimated.

It is generally accepted that more extreme and variable weather can be expected as a result of climate change (although substantial uncertainty remains in the prediction of the changes). This can cause hazards to structural safety in different forms including changes in loads (extreme winds, extreme precipitation leading to flooding, extreme deposition of ice on structures and extreme diurnal temperature changes) and material behaviour (SCOSS 2000). To counter such adverse trends, emphasis needs to be given on providing additional degree of robustness in the design of new structures and on the identification of the vulnerable parts of existing structures.

4 STRUCTURAL RISK

The exploitation of vulnerabilities knowingly or unknowingly could cause grave harm to the structure, occupants and possibly to the much bigger system. Vulnerability together with threat can produce high risks. So long as there is no undesirable action to exploit the vulnerability, it is only a potential risk. However, if a threat is active, the combination of the active threat and the vulnerability creates a real risk to the structure and its users.

Hence, after having identified the different failure scenarios, it is important to consider the probability of actions and the likelihood of damage occurring. A consideration of context is also required. For example, for a building to be used as a theater acceptable risk level will be lower than when it to be used as a warehouse.

With the distinction between the damage due to an action and consequence of a damage, risk assessment requires a slightly different approach. It is necessary to define the probability of damage due to an action (or a combination of actions) and then the probability of that damage leading to the consequences. The latter depends upon the actions present after the damage. For example if the stress levels were already high before the damage, self-weight of the members alone may lead to failure. The former is rather complicated because of many uncertainties and perhaps best captured using an evidence-based approach. Figure 2 helps to illustrate the risk as a function of vulnerability and probability of damage.

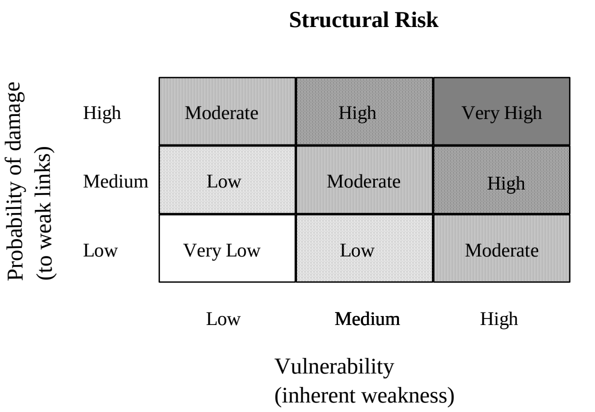

Figure 2. Structural risk as a function of vulnerability and probability of damage

Once vulnerable members have been identified and the associated risks assessed, these results can be used to develop an action plan. Parts of a structure that are found to be highly vulnerable or pose high risks may need to be modified. Modifications may take different forms. Structural consequences may be reduced, for example, by providing more connectivity through installation of additional members. Effort required to damage a member can be increased, for example, by strengthening the member or replacing it with a stronger member. Parts of the structure where modifications are either difficult or cannot be justified because of associated costs, may need to be regularly monitored and protected from threats. Thus addressing potential vulnerabilities in a structure requires a flexible and evolutionary approach.

5 AN EXAMPLE

Figure 3a shows a single storey frame where all of the joints are modeled as ball-joints. Members (m0 to m11) have the same cross-sectional and material properties for the ease of illustration. Connections between supports are referred as reference members (r12 to r17). This frame can be built by joining together two tetrahedrons - one formed by nodes 0,1,3,4 and the other by nodes 1,2,3,6. The rest of the structure can be obtained by adding a node with three members successively. For this simple structure it is easy to notice that any damage to a member forming any of two tetrahedrons will cause severe consequences. This can be avoided by having a bracing at the top face. Vulnerability analysis is able to demonstrate this, identify other critical members and to guide where additional members should be provided to limit the failure. Table 1 summarizes the results obtained using the vulnerability analysis program for different damage events.

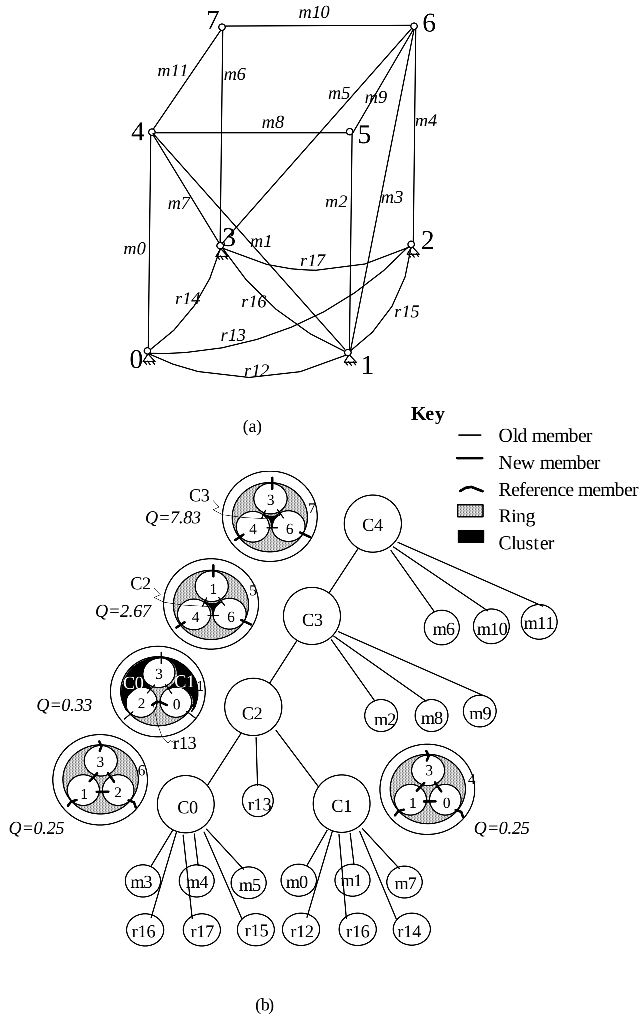

Figure 3. (a) An example structure and (b) its hierarchical model (along with graphical description of clusters)

Failure of member m1 due to any action (examined subsequently) would cause near total collapse of the structure. However this consequence can be reduced by placing a new member between nodes 4 and 6 or between nodes 5 and 7. Failure of member m2 also causes significant loss of structure. This loss can also be reduced by a member either between the nodes 4 and 6 or between the nodes 5 and 7. However, the latter is to be preferred because of more reduction in the consequence should damage occur to member m2.

Vulnerability analysis results can be combined with probability of damage to get a measure of structural risk. For the purpose of illustration, only a qualitative analysis of structural risk is presented considering four actions. The actions considered include gravity load on top, lateral load, vehicle impact and corrosion. Effects of these actions and estimated probabilities of damage to members (member m0, m1 and m2 in particular) are summarized in Table 2. Because of the identical properties of the members, a member carrying higher force is assigned a higher probability of damage. For vehicle impact, member m0 is given a higher probability of damage than member m1 or m2 alone. The latter two members are close to each other and may share the impact. Degradation due to corrosion is assumed to be more near the supports. In practice, this information will come from condition assessments. Overall probability of damage has been obtained by giving equal importance to all of the actions. However, further work is in progress using Interval Probability Theory (Cui *et al* 1990).

An assessment of structural risk is made by mapping the results of vulnerability analysis and the probability of damage onto Figure 2. It can be seen that structural risk is 'high' for damage to members m0 and m1 while it is 'moderate' for member m2. However, by providing an additional diagonal member at the top, structural risk due to members m0 and m1 can be reduced to 'moderate' level. Alternatively, risk can be reduced by controlling the actions (e.g. vehicle impact for member m0) which give rise to high probability of damage.

Table 1. Summary of vulnerability analysis results*

Damage	Structure as shown			Structure with new member between nodes 4, 6			Structure with new member between nodes 5, 7		
	D	C	V.I.	D	C	V.I.	D	C	V.I.
m0	0.06	0.97	15.5	0.06	0.14	2.5	0.06	0.08	1.4
m1	0.04	0.97	22.0	0.04	0.10	2.5	0.04	0.10	2.5
m2	0.06	0.66	10.6	0.06	0.55	9.3	0.06	0.20	3.4
m1 + m3	0.09	1.0	11.3	0.08	1.0	12.0	0.08	1.0	12.0

* Notation: D - Damage demand, C - Consequence, V.I. - Vulnerability Index (low <10, medium 10 -16, high >16)

Table 2. Probability of damage to members due to different actions

Member	*Gravity load*	*Lateral load*	*Vehicle impact*	*Corrosion*	Overall probability
	Equal load applied to nodes 4, 5, 6 and 7. Members m0, m2, m4 and m6 stressed. No force in other members	Equal load applied to nodes 4 and 7. Members m1 and m5 more stressed than m0, m4 or m10. No force in m2, m8, m9 and m11	Vehicle approaching with the same momentum	Identical conditions near the supports	
m0	High	Medium	High	Low	High
m1	Low	High	Medium	Low	Medium
m2	High	Low	Medium	Low	Medium

6 CONCLUSION

The purpose of the vulnerability theory is to gain an understanding of the quality of the form of a structural system.

Vulnerability analysis is concerned with the identification of vulnerable failure scenarios. Damage to these elements may lead to disproportionate failure.

All actions which may cause damage to vulnerable elements should be analyzed for the risks they pose to the structure.

For a new structure vulnerable elements can be redesigned. However, in existing structures these elements may either be strengthened or protected through different measures until risk is acceptably low.

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