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Industry CPD

Calculating cracking deformation in concrete

This CPD module, sponsored by Buildsoft, explores the benefits of advanced structural analysis software when calculating cracked deflections of reinforced concrete beams and floor slabs.

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1 hour of verifiable CPD

DEFINITIONS

The **initial shape** of a concrete beam or floor slab is the reference line (reference plane) of the loadbearing element in the unloaded condition.

The **deflection** is the displacement of the reference line (reference plane) under the influence of the applied loads compared with the initial shape.

The **instantaneous deflection** due to applied loads is the part of the deflection that occurs almost instantaneously after applying the loads.

The **time-dependent deflection** due to applied loads is the part of the deflection that occurs due to concrete creep after applying loads that are assumed to be constant through time.

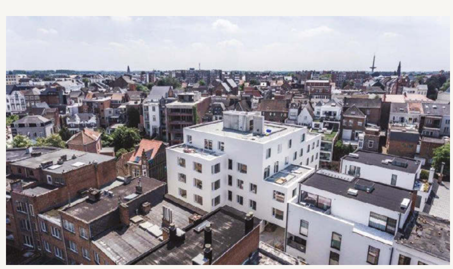
The **additional deflection** from the point in time t_i is the part of the deflection that occurs after t_i. It consists of both:

- the remaining part of the time-dependent deflection under the loads that have already been applied at the point in time t_i
- the instantaneous and time-dependent deflection under the loads applied after the point in time t_i.

Introduction

Thanks to a better understanding of material properties and advanced analysis tools, structural engineers are able to achieve more lightweight designs. As a result, comfort requirements are becoming increasingly important. For concrete beams and floors, maximum deflection often becomes the decisive criterion. But this raises new questions, such as: 'How should deflections be calculated?' and 'What are the limit values for maximum deflections?'

Calculating deflections of reinforced concrete beams and floor slabs is a complex matter because the Young's modulus of concrete evolves over time. In addition, when cracking is involved, the moment of inertia of a cross-section is far from constant along the length of a beam or floor slab. Advanced structural analysis software is, therefore, indispensable. This article discusses how to deal with those complex conditions in practice, according to EN 1992-1-1.



↑FIGURE 1: Fonteynbrug Residence project in which Buildsoft programs were used to calculate deflection over time (architect: Stéphane Beel Architects; engineer: Structural engineering office Concreet; contractor: Algemene Bouwonderneming Hooyberghs)

Calculating the cracked deflection

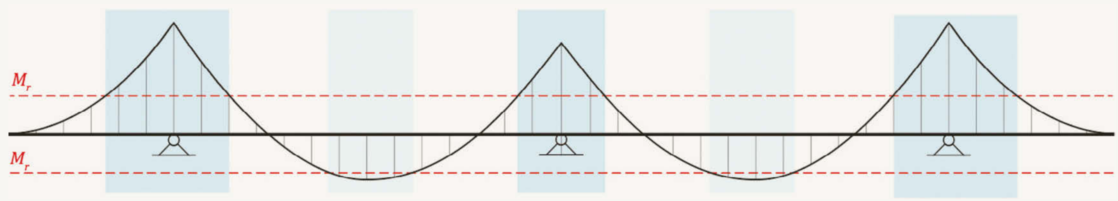
The elastic deflection of reinforced concrete beams or floor slabs will only match the actual deflection if the beam or floor slab's cracking moment is not exceeded. Because of the low tensile strength of concrete, this situation will usually apply in practice. As soon as stresses exceed the concrete's tensile strength, cracks will appear, and the cross-section's stiffness will significantly decrease, resulting in more considerable deflections.

To calculate the deflection of reinforced concrete beams or floor slabs, we need to consider concrete cracking. The degree to which the concrete is cracked will depend not only on its tensile strength, but also on the loads and the actual amount of reinforcement.

The calculation method discussed hereafter (available in the Diamonds structural analysis software) is a logical extension of design rules for structural elements (such as beams) bearing loads in one direction only. After discussing its principles for the case of unidirectional loadbearing elements, we'll examine how we can extend the scope of this calculation method to structural elements bearing loads in two directions.

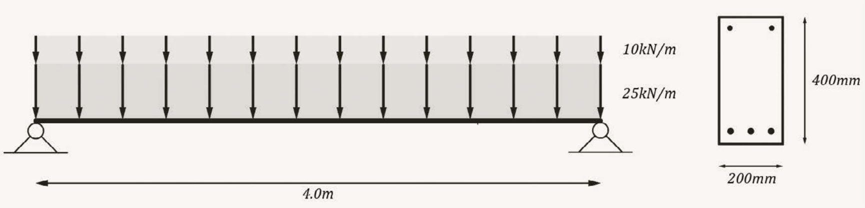
For elements subject to bending, **Equation 1** applies for the calculation of the vertical deflection δ:

$$\frac{d^2\delta}{dx^2} = \frac{1}{r} = \frac{M}{EI} \qquad (1)$$

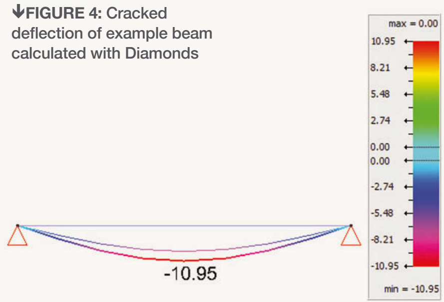


←**FIGURE 2:** Distribution of cracked zones in continuous beam

↓**FIGURE 3:** Example beam



↓**FIGURE 4:** Cracked deflection of example beam calculated with Diamonds



For reinforced concrete elements, the curvature value $1/r$ depends on whether or not the cross-section is cracked. A cross-section is cracked only when the bending moment exceeds the cracking moment M_r. The cracking moment M_r is determined by

$$M_r = f_{ctm,fl} \cdot W \tag{2}$$

in which:

$f_{ctm,fl}$ the mean flexural tensile strength of concrete
W is the moment of resistance of the uncracked fictitious cross-section that consists of the entire concrete cross-section augmented by α (= E_s / E_c) times the section of the reinforcement bars.

For a cross-section in uncracked zones of the beam (for which $M < M_r$), the local curvature $1/r_1$ is calculated from:

$$\frac{1}{r_1} = \frac{M}{EI_1} \tag{3}$$

in which:

E is the Young's modulus of concrete
I_1 is the moment of inertia of the uncracked fictitious cross-section.

For a fully cracked section (for which $M > M_r$), the local curvature $1/r_2$ is calculated from:

$$\frac{1}{r_2} = \frac{M}{EI_2} \tag{4}$$

in which:

E is the Young's modulus of concrete
I_2 is the moment of inertia of the cracked fictitious cross-section, consisting only of the compressed concrete cross-section augmented by α times the section of the reinforcement bars ($\alpha = E_s / E_c$).

In **Equations 3 and 4** creep is accounted for using the creep factor ϕ ($E = E_c / 1 + \phi$).

Figure 2 shows the distribution of cracked zones in a continuous beam. In the white zones, the bending moment remains smaller than the cracking moment M_r. The beam is not cracked in those zones. In the light blue zones, the bending moment exceeds the cracking moment M_r a little. While in the darker blue zones, the cracking moment M_r is exceeded by a lot more. The light blue zones will be less cracked than the darker blue zones. So, we need an expression that takes the amount of cracking into account: the average local curvature $1/r$ for a cross-section in a cracked zone (for which $M > M_r$) is calculated as the weighted average between the uncracked and cracked curvature:

$$\frac{1}{r} = (1-x)\frac{1}{r_1} + x\frac{1}{r_2} \tag{5}$$

in which:

x is the distribution coefficient equal to $x = 1–0.5\,(M_r / M)^2$.

Rather than calculating the deflection through a double integration of the local curvatures $1/r$, we can divide the beam into several elements with variable stiffness EI, replacing I with I_1 for the uncracked zones and deriving I for cracked zones from **Equation 5** in which curvatures $1/r$ are replaced by M/EI.

We can easily extend the latter method to structural elements bearing loads in two directions. For each node of a finite element mesh, the moment M and the cracking moment M_r can be determined for both reinforcement directions. **Equations 5 and 6** then provide a way to estimate the stiffness EI for both directions.

See the **Worked example** for a demonstration of this calculation.

Calculating additional deflection

Determining the additional deflection requires the time at which the loads act. The deflection as a function of time can be calculated using superposition:

→| For each load group:
  a) calculate the instant cracked deflection using **Equations 1 to 5**
  b) calculate the delayed deflection due to creep.

→| At each step in time, take the sum of:
  a) the instant cracked deflection of the loads that are present
  b) the appropriate part of the delayed deflection due to creep.

But this method comes with a few difficulties:

→| Cracking is non-elastic material behaviour. It does not allow superposition.
→| The inertia of cracked cross-sections must be calculated instantaneously.
→| The creep factor for a given load depends on the age of the concrete at the time the load is applied: the older the concrete, the lower the creep factor for that load.

The following approach can be suggested (as implemented in Diamonds):

→| The superposition principle is applied by

WORKED EXAMPLE: CALCULATING THE CRACKED DEFLECTION

We use the beam below to illustrate the method for cracked deformation.

Data

C25/30, S500, $A_{s,top}$ = 226mm², $A_{s,bottom}$ = 942mm², $d_1 = d_2$ = 40mm, ϕ = 2
25kN/m dead load and 10kN/m live load resulting in $M_{ULS\ FC}$ = 102.9kNm, $M_{SLS\ RC}$ = 74kNm

Calculation

Determine the cracking moment M_r

$$M_r = f_{ctm.fl} \cdot W = 19.51\text{kNm}$$

where:

$$f_{ctm.fl} = 3.08\text{MPa}$$

$$W = 6337\text{cm}^3$$

The distribution coefficient x equals

$$x = 1 - 0.5\left(\frac{M_r}{M_{SLS\ RC}}\right)^2 = 0.965$$

The deflection of an uncracked section δ_1 equals

$$\delta_1 = \frac{5}{384}\frac{8M_{SLS\ RC}\ L^2}{E_c I_1} = 3.21\text{mm}$$

where:

$$E_c = 31476\text{MPa}$$

$$I_1 = 1222\,484\text{cm}^4$$

The deflection of a fully cracked section δ_2 equals

$$\delta_2 = \frac{5}{384}\frac{8M_{SLS\ RC}\ L^2}{E_c I_2} = 11.25\text{mm}$$

where:

$$E_c = 10\,492\text{MPa}$$

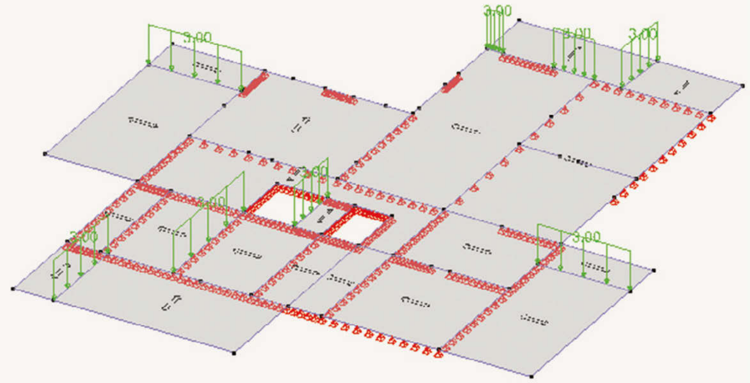
$$I_2 = 104\,513\text{cm}^4$$

The maximum cracked deflection of the beam equals

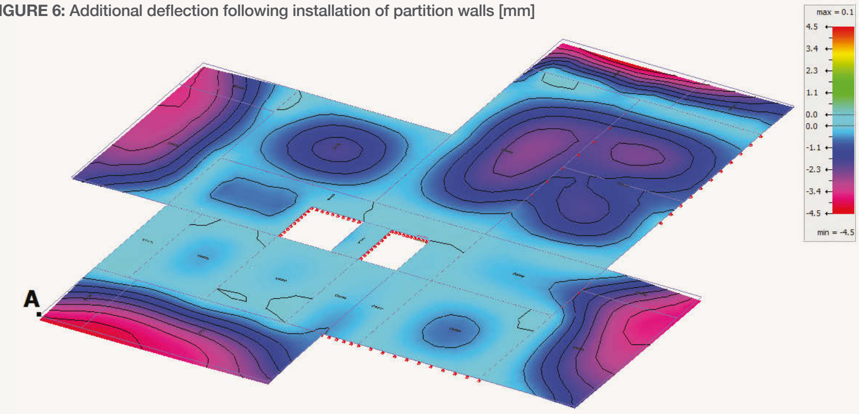
$$\begin{aligned}\delta &= (1-x)\delta_1 + x\delta_2 \\ &= (1-0.965)\cdot 3.21\text{mm} + 0.965 \cdot 11.25\text{mm} \\ &= 10.97\text{mm}\end{aligned}$$

Figure 4 shows the example beam calculated with Diamonds. Both give similar results.

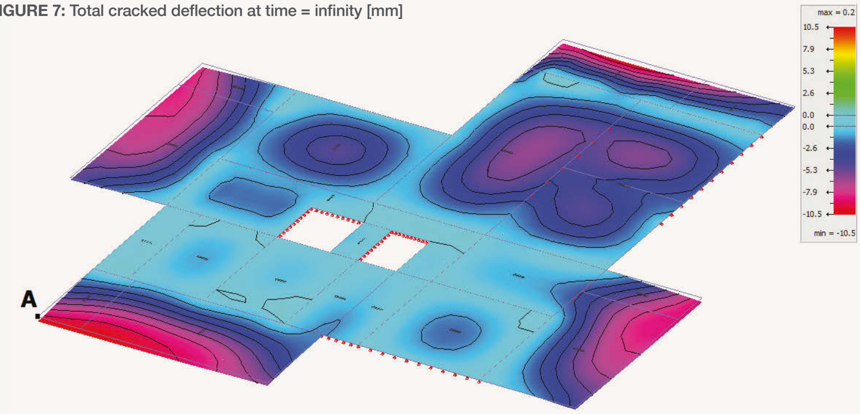
↓FIGURE 5: Diamonds geometry model of one floor slab from building in Figure 1, showing loads due to partition walls



↓FIGURE 6: Additional deflection following installation of partition walls [mm]



↓FIGURE 7: Total cracked deflection at time = infinity [mm]



↓FIGURE 8: Elastic deformation [mm]

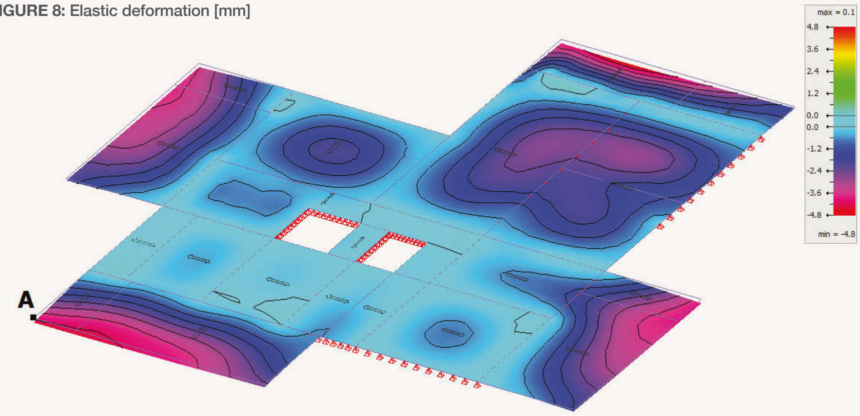


Table 1: Maximum deflection as a function of time in point A

Time [days]	Deflection [mm]
t = –28	0.0
t = +28	–2.5
t = –60	–4.4
t = +60	–5.2
t = –90	–6.2
t = +90	–6.2
t = –120	–6.6
t = +120 RC	–7.3
t = inf. RC	–10.5 (see Fig. 7)

specifying which load combination should be considered decisive for concrete cracking.

- This can be done separately for each load group (the amount of cracking is deemed to be constant through time for each load group). Usually, a characteristic load combination is recommended for live loads, and a quasi-permanent load combination for dead loads.
- The creep factor is constant for each concrete grade and for all load cases.

Calculating additional deflection: practical example in Diamonds

The above approach is illustrated through a practical example. **Figure 5** shows the geometry of a floor slab. It is assumed that the self-weight acts at 28 days, the dead loads at 60 days, the partition walls at 90 days (their distribution is shown in **Fig. 5**) and the live loads at 120 days.

We would like to calculate the additional deflection following the installation of the partition walls. Practically, this means we must calculate the difference between the total deformation in the characteristic load combination (at time infinity) and the deflection present just before the partition walls are placed. Because the additional deflection is more damage-related than comfort-related, we use the characteristic load combination. Imposing this restriction for quasi-permanent load combinations would be insufficient.

Table 1 shows the increase in deflection over time in point A, while **Figure 6** shows the calculated values for the additional deflection.

In addition to calculating the additional deflection, this method can also be used to calculate the total deflection (**Figure 7**). If we compare the total cracked deflection with the elastic deflection (**Figure 8**), we notice a factor 2 difference, which is not uncommon. The cracked deflection can be up to a factor of 5 higher than the elastic deformation.

Complying with design codes

Now that we have calculated the maximum deflection, to which value should it be limited? There is a distinction between the total deflection and the additional deflection.

- The **total deflection** is generally limited to the span length divided by 250 (or 1/125 for a cantilever) in an SLS QP (self-weight, permanent loads and approx. 30% of the service loads). By applying a counter-deformation, the deflection can be fully or partially compensated.
- The **additional deflection** is generally limited to the span length divided by 500 (or 1/250 for a cantilever) where damage to partition walls is to be avoided.

Conclusion

The non-linear material behaviour of concrete requires the calculation of deflection to consider cracking and creep. Advanced structural analysis software is vital to estimate the total and additional deflections for 2D and 3D models.

Questions

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1) What knowledge is essential to estimate the cracked deflection? (Select all possible answers)

☐ Position of the stirrups
☐ Bending moment diagram
☐ Concrete grade
☐ Steel grade

2) Which statement is true about the cracking moment M_r?

☐ When the bending moment in a beam exceeds the cracking moment M_r, the beam is fully cracked
☐ The cracking moment M_r is the bending moment in a concrete beam or floor slab just before the tensile zone starts to crack
☐ When the bending moment in a beam exceeds the cracking moment M_r, the curvature $1/r$ remains constant
☐ When the bending moment in a beam exceeds the cracking moment M_r in one spot, the entire element is cracked

3) Calculate the elastic deformation of the example beam in Fig. 3. Use the formula $\delta = \frac{5}{384}\frac{8M_{\mathrm{SLS\,RC}}\,L^2}{E_c I}$ where E_c = 31 476MPa and the moment of inertia I based on a pure concrete cross-section. Which statement is true about this elastic deformation?

☐ The elastic deformation equals the uncracked deformation δ_1
☐ The elastic deformation is larger than the uncracked deformation δ_1
☐ The elastic deformation is smaller than the uncracked deformation δ_1
☐ It is impossible to predict the elastic behaviour of the beam

4) What is the strictest requirement that the additional deflection in Fig. 6 satisfies? Use a span of 3m.

☐ L/250
☐ L/300
☐ L/500
☐ L/1000

5) If the creep factor ϕ = 2, how much larger is the deflection after creep compared with the instantaneous uncracked (elastic) deflection? The elastic deflection ratio with creep/without creep is:

☐ 0.5
☐ 1 (equal)
☐ 2
☐ 3

6) Following on from the previous question, if the section (and therefore deflection) were cracked, would the ratio with creep/without creep, compared with the non-cracked elastic ratio, be:

☐ Smaller
☐ Equal
☐ Larger