Extreme dynamic loading by humans on footbridges

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Introduction

When crossing a footbridge, the largest dynamic load a person can generate is through running or jumping. Running is much more likely especially as running has become increasingly popular over the last few years, especially since the start of the COVID-19 pandemic. With this comes the inevitability that these runners will run across a bridge either in a race or on a training run, causing the risk that the bridge will react differently to a running user than a walking user. This would not only affect the speed that the runner can travel but could also cause long-term serviceability issues for the bridge.

The European standards of structural design do not consider running within normal use, and only suggest a group of joggers 'may cross a footbridge with a frequency of 3Hz'. This is incorrect as shown by the large differences in figure 3, comparing the force produced when running, walking and speed-walking.

This over-simplification of the Eurocodes assumes that people will only ever cross a bridge walking or jogging very slowly, therefore a better method of modelling the frequency and force produced by a runner is required.

Therefore, a parametric study was conducted to assess vibrations produced by a runner so it can be seen how a footbridge of given metrics would react, using a MATLAB script summarized in figure 1. The running metrics would be systematically changed so that the metric with the greatest effect can be found.

Aims and objectives

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The aim of this project is to discover which running metric causes the greatest effect on the acceleration of a footbridge. To achieve this aim, the following objectives will be explored:

1. To model the force exerted on the ground by a person running.
2. To apply this force to a bridge with given metrics, obtaining the resultant acceleration of the bridge.
3. To systematically change the running metrics in the model and compare resultant accelerations.

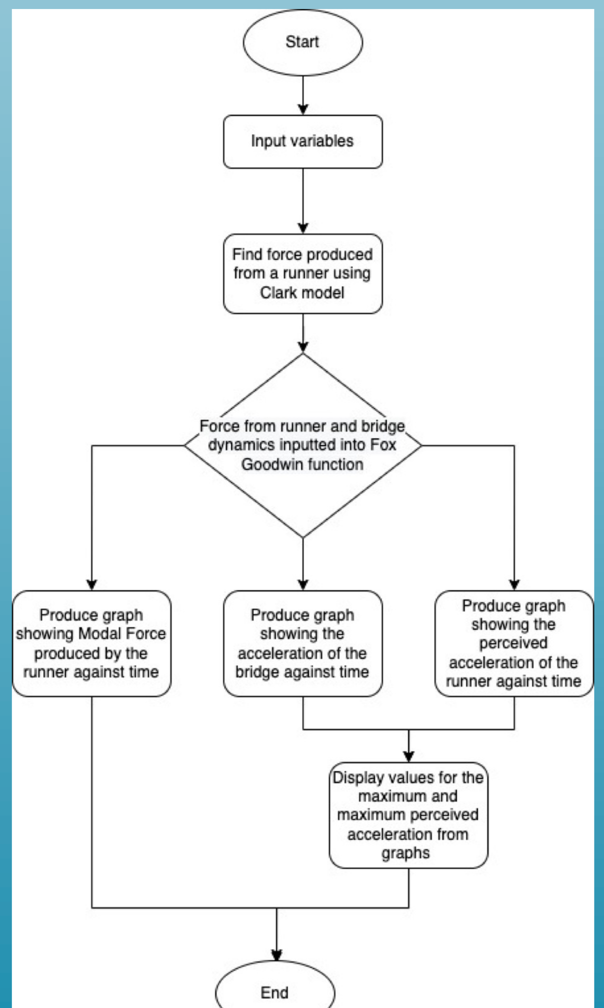

Figure 1: A flow diagram of the code, created in the planning stage

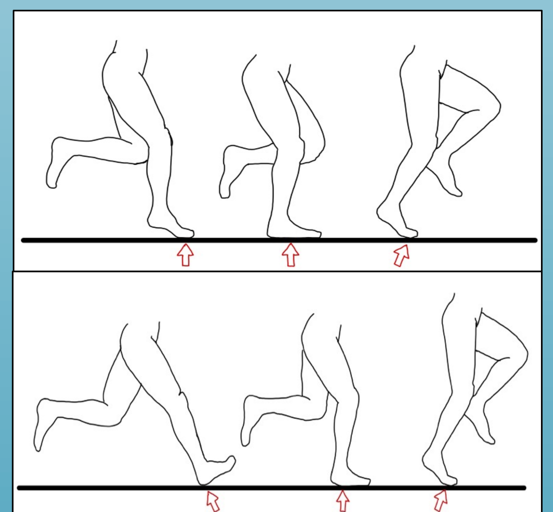

Figure 2: A forefoot and rear foot strike pattern

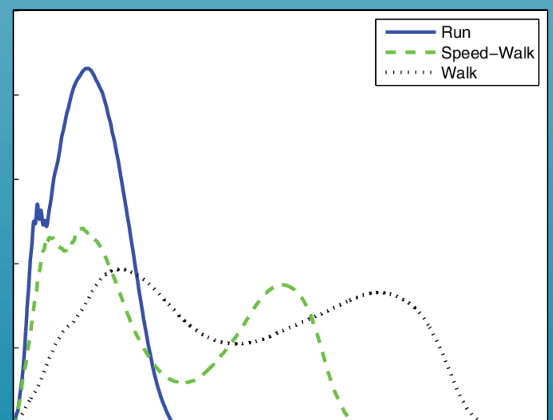

Figure 3: Force produced in a single step running, speed-walking and walking

Developing the model

To find the force produced by a runner, the Clark model (3) was used to find the force produced by a single step. The mass of the runner is split into m1 and m2 using the percentages shown in figure 4.

The values of the change in vertical velocity of m1 (dv1) and the time interval for dv1 (dt1) are chosen based on the speed and foot strike style (shown in figure 2) of the runner (4). These values are shown in table 1.

	Rear foot strike		Forefoot strike	
Running speed	dv1	dt1	dv1	dt1
Slow (3-4 mph)	0.850	0.029	1.300	0.046
Medium (5-6 mph)	1.280	0.023	1.370	0.034
Fast (7 mph)	2.010	0.019	2.080	0.027

Table 1: the 'dv1' and 'dt1' values for various running speeds and foot strike patterns

The values for F1 and F2, the force produced by the masses m1 and m2 are:

$$F_1(t) = \frac{A_1}{2}\left[1 + \cos\left(\frac{t-B_1}{C_1}\pi\right)\right]$$
$$F_2(t) = \frac{A_2}{2}\left[1 + \cos\left(\frac{t-B_2}{C_2}\pi\right)\right]$$

Where:

$$A_x = 2 * F_{x,avg}$$
$$B_1 = C_1 = dt_1$$
$$B_2 = C_2 = 0.5 * t_c$$
$$F_{1,avg} = \frac{J_1}{2 * \Delta t_1}$$
$$F_{2,avg} = \frac{J_2}{t_c}$$
$$J_1 = (m_1 * \frac{\Delta v_1}{\Delta t_1} + m_1 * g)(2 * \Delta t_1)$$
$$J_T = m_b * g * \frac{t_{step}}{t_c}$$

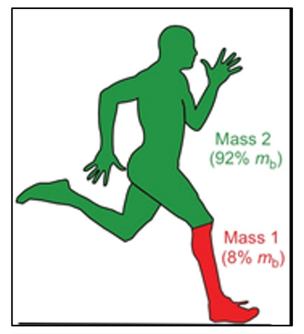

Figure 4: A diagram showing the weight distributions of a runner in the Clark model (3)

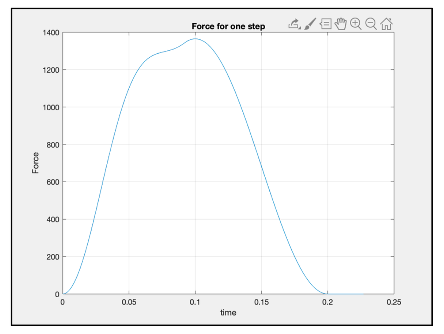

Figure 5: The force produced by a single step plotted against time

The frequency of the runner is found using:

$$Frequency = \frac{Speed}{Stride\ length}$$

Figure 5 shows the resultant graph when F1 and F2 are added together. This graph is then repeated for the number of steps it takes to cross the given bridge span and then multiplied by the natural mode (half sine) to obtain the modal force produced by the runner. This graph is shown in figure 6.

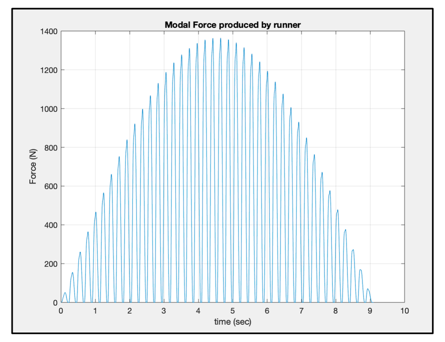

Figure 6: The modal force produced by the runner

The Fox-Goodwin algorithm is then used to obtain the acceleration of the mid-point and the perceived acceleration as the runner crosses the bridge. The inputs to this are:

- The modal mass of the bridge (M)
- The stiffness coefficient (K)
  - $\Omega = 2 * \pi * fn$
  - $K = M * \Omega^2$
- The damping coefficient (C)
  - $C = 2 * \zeta * \sqrt{(K * M)}$
- The modal force produced by the runner
- The time interval

The acceleration is the plotted against time (shown in figure 7).

The perceived acceleration is found by multiplying the graph in figure 7 by the natural mode (shown in figure 8).

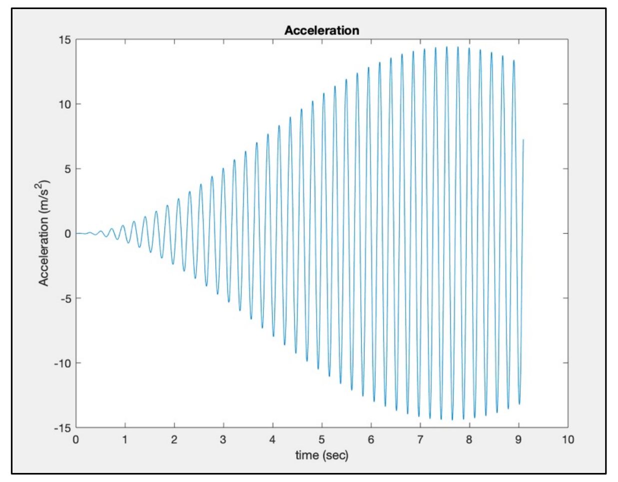

Figure 7: Acceleration of the midpoint as the runner crosses

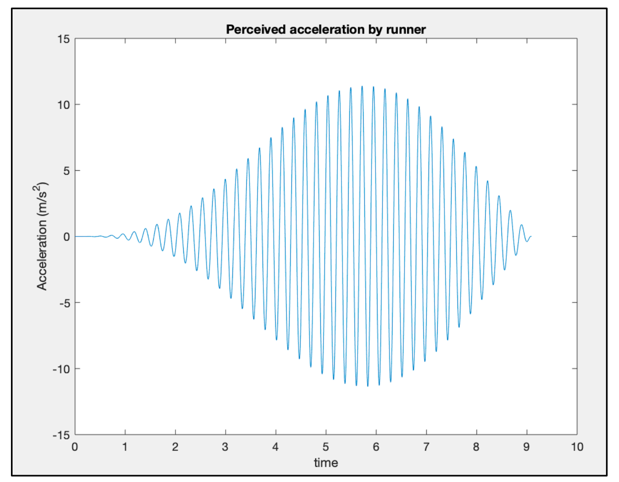

Figure 8: Acceleration felt by the runner as they cross the bridge

Method

To which running metric causes the greatest effect on the acceleration of a footbridge, the code was systematically run changing one running metric at a time and keeping the bridge metrics the same. The bridge metrics that were used are:

- Length = 50m
- Physical mass = 5000kg
- Modal mass = 2500kg
- Damping ratio = 0.5%
- Frequency = 4 Hz

These values were chosen as they are typical of a conventional footbridge (as per values found in literature, shown in figure 9.

The running metrics that were changed are:

- Mass- 60, 70 and 80 kg
- Running speed- 3.5, 5.5 and 7.5 m/s
- Stride length- 1, 1.25 and 1.5 m
- Ground contact time- 0.25, 0.2 and 0.15 s
- Foot strike pattern- rear and forefoot strike patterns, each with three speeds investigated.

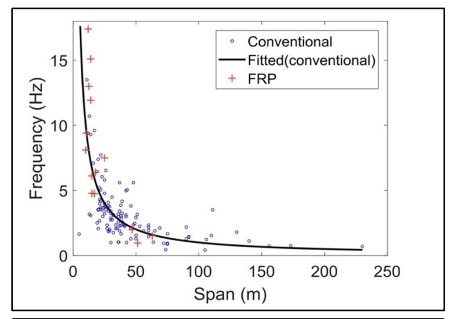

Figure 9: frequencies found in different length FRP and conventional footbridges (6)

When each metric was investigated, the median value was used for the others.

The simulations were each run three times to ensure that human or machine error does not affect results.

The acceleration values can be compared to values found in literature to establish the affect they would have on a person. This was compared using graphs showing the human sensitivity to vibration from ISO standards (1)(2) shown in figure 10.

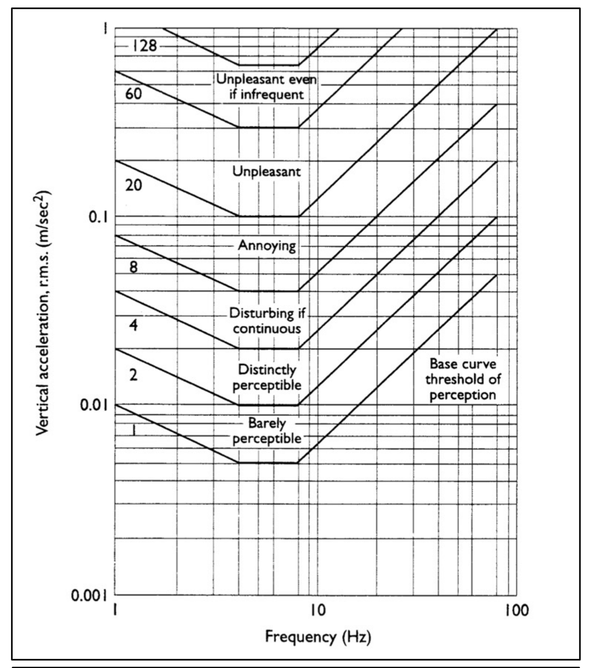

Figure 10: Human sensitivity to acceleration of a structure

Results

Running metric	Value	Frequency (Hz)	Maximum Acceleration (m/s^2)	Maximum Perceived Acceleration (m/s^2)
Mass (Kg)	60	4.4	1.5468	1.5451
	70	4.4	1.8046	1.8027
	80	4.4	2.0624	2.0602
Speed (m/s)	3.5	2.8	0.7857	0.7842
	5.5	4.4	1.8046	1.8027
	7.5	6	0.224	0.2237
Stride length (m)	1	5.5	0.3506	0.3504
	1.25	4.4	1.8046	1.7234
	1.5	3.6667	2.8154	2.6383
Ground contact time (sec)	0.15	4.4	2.6886	2.6846
	0.2	4.4	1.8046	1.8027
	0.25	4.4	1.0858	1.085
Rear foot strike (m/s)	3.5	2.8	0.7729	0.7715
	5.5	4.4	1.6572	1.6554
	7.5	6	0.2609	0.2609
Forefoot strike (m/s)	3.5	2.8	0.7857	0.7842
	5.5	4.4	1.7043	1.7025
	7.5	6	0.182	0.182

Table 2: Results of simulations

Analysis and Conclusions

The results in table 2 show that:

- The combination of speed and stride length (the factors that determine the frequency of the runner) cause the greatest change in response from the bridge.
- When the frequency was kept constant, the ground contact time of the runner caused the largest acceleration of the bridge.
- The acceleration produced can be decreased by the runner changing their running style or speed. This would be at their detriment, as it would mean they are sacrificing either energy or time.
- Increasing the mass increases the acceleration of the footbridge
- A Forefoot strike causes a greater acceleration than a rear foot strike

Only a few of the results are less than the maximum recommended acceleration from literature, shown in figure 10.

An 'unpleasant, even if infrequent' user experience would be noted with an increased probability of synchronization (5), which would likely cause the runner to negatively impact their running technique and economy in order to mitigate this.

When comparing these values in table 2 to the values in literature, it shows that the hypothetical footbridge based on the chosen bridge metrics would not be suitable for users to run across in almost all cases. However, when using the suggested values for the frequency of a 'group of joggers' from the Eurocodes, it would be very likely be acceptable, as the frequency is far from the resonant frequency of this footbridge, causing a smaller response.

These are significant findings when considering the lack of research papers looking into running as a source of vibration for bridges and the over-simplification of the frequency of a runner in Eurocodes. These findings should encourage engineers to reconsider the loads and frequencies they use in the design of their bridges.

References

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