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Industry CPD

Nonlinear finite element modelling and analysis

This CPD module, sponsored by Oasys, examines the use nonlinear analyses, which are set to become a central option in the design workflow under the second generation of Eurocodes. It explores common misunderstandings, ways that nonlinear analysis differs from linear, and special considerations needed to get the right answers.

Continuing professional development (CPD) ensures you remain competent in your profession. Chartered, Associate and Technician members of the Institution must complete a specified amount each year. All CPD undertaken must be reported to the Institution annually. Reading and reflecting on this article by correctly answering the questions at the end is advocated to be:

1 hour of verifiable CPD

"Using a term like nonlinear science is like referring to the bulk of zoology as the study of non-elephant animals"

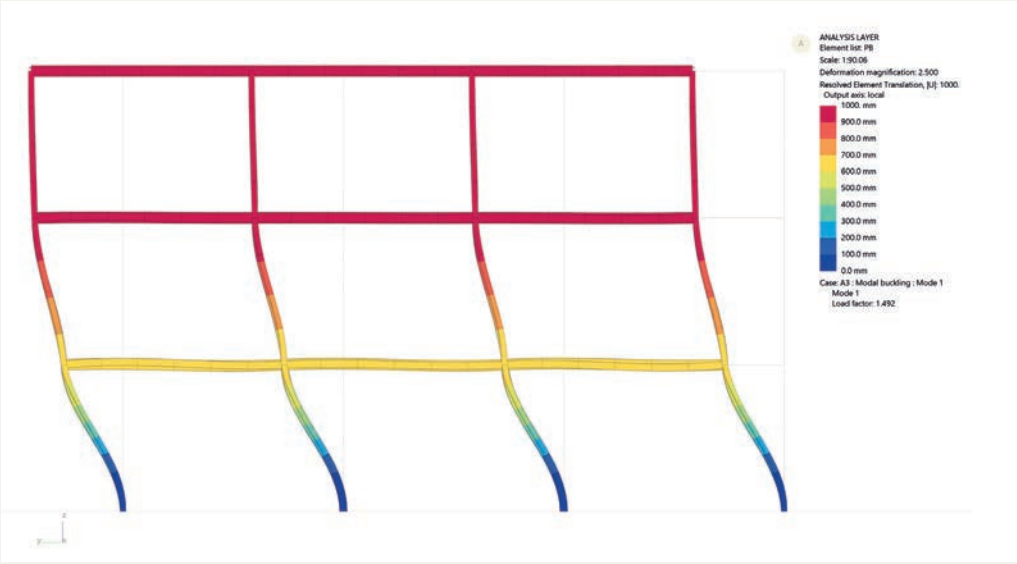
Attributed to mathematician Stanisław Ulam

Introduction

We are taught linear approximations to engineering problems and, in forgetting that they are approximations, we are fooled into thinking that everything is linear. Even saying 'linear' and 'non-linear' implies that linear is the norm; but as our structures exist in the real world, the bulk of structural behaviour is actually nonlinear. This means that there are times *when linear analyses are insufficiently accurate and you must take nonlinear effects into account*[1].

Understanding when to use nonlinear analyses, as well as their special requirements, is becoming increasingly important: the new editions of Eurocodes 2 (EC2) and 3 (EC3)[2,3] are putting nonlinear analysis as a central option in the design workflow. But there are common misunderstandings about nonlinear analysis: some see it as being too difficult and specialised, while others use it while being ignorant that it differs from linear. It is important to know that nonlinear analyses are perhaps something that you are already using, but also that they require special considerations to get the right answers.

As we explored in our last CPD article[4], the linear approach to analysis does have many advantages, including that it provides equations that are directly solvable. With tacit assumptions about load directions, small deflections and ignoring buckling or yielding, we can get useful answers – provided that those assumptions are, indeed, valid. But nonlinear behaviour is there in our everyday work: columns buckle, steel yields, braces tense, masonry stands and concrete cracks. In a linear material, doubling the load also doubles the stress, strain and deflection. Linear structures behave in the same way if the load reverses. But in a nonlinear material, structure or state, these assumptions do not hold. Carbon steel has a linear stress–strain relationship until it yields; a steel beam deflects linearly but a steel cable does not; a CHS brace works in tension and compression but steel flats only in tension.



↗**FIGURE 1:** Sway global buckling mode

So how should we work with the nonlinear reality?

Types of nonlinearities

First, we need to recognise that there are several types and degrees of nonlinearity. These include directional/contact, material, and geometric, which can be used together or separately as needed. Like linear analyses, nonlinear analyses also include assumptions that you need to be aware of to create a valid model.

Geometric (second-order) nonlinearity

Geometric nonlinearity comes about from large displacements. In linear (first-order) analysis we assume that the equilibrium is found at the starting geometry, meaning that we assume zero displacement, but then calculate what the displacement will be. This is, of course, a contradiction and is only acceptable when the deflections are small.

An important aspect of geometric nonlinearity is that *axial loads will change the member stiffnesses*: reducing them with compression and increasing with tension*. If using a p-delta method of analysis, these 'geometric stiffnesses' are calculated from the forces in an initial static linear analysis based on the regular stiffnesses. These two stiffnesses are then added together and the system reanalysed to get a more accurate result. A full nonlinear analysis solver, on the other hand, iterates until the equilibrium geometry is found, updating the structural stiffnesses as it goes, allowing large displacement analyses such as snap through.

The effects of these large displacements mean that *additional moments are generated, which magnify as the deflection increases* and stiffness of the structure reduces, potentially increasing the deflections and hence moments even more **(Figure 1)**.

Lightweight structures such as cable nets and fabrics typically need large deflections to achieve equilibrium. In the example shown in **Figure 2**, the initially straight cable† deflects down until the axial forces induced in the cable combined with the new angle of the cable achieve equilibrium. Note that *there are many positions where equilibrium is possible*; these depend on the load and span, axial stiffness of the cable, stiffness of the supports, and any pretension in the cable.

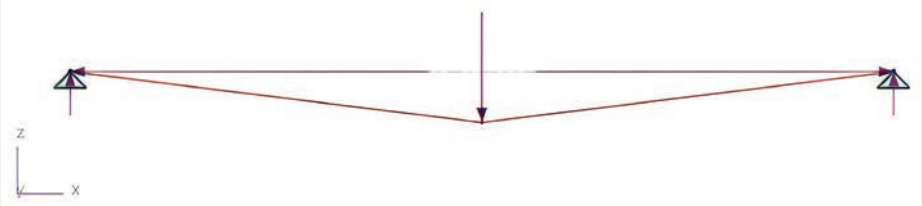
As the stiffness of a cable and fabric structures are highly dependent on the combination of geometry and pretension, finding that geometry is an essential step in the design process. This requires a type of analysis called *form-finding*, which updates the model geometry to one where the permanent forces are in equilibrium. Once the form is found, static nonlinear analysis with the additional variable loads can continue as normal **(Figure 3)**.

Material nonlinearity

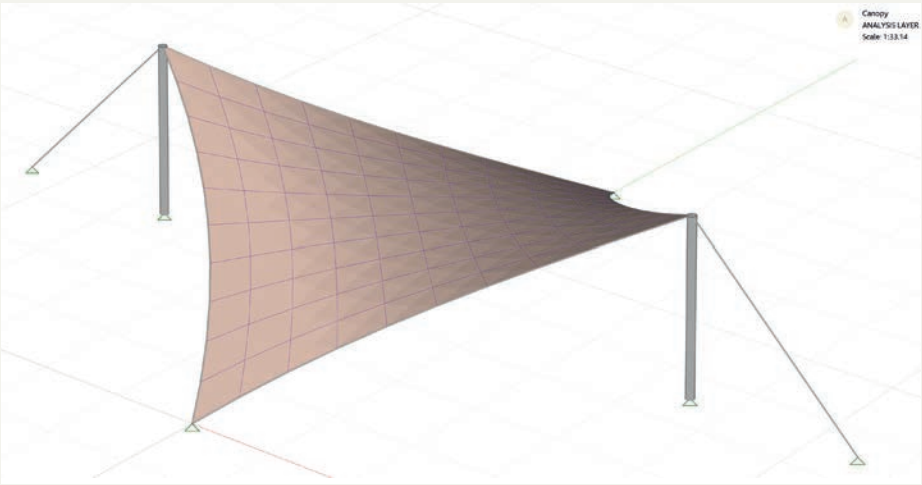
Material nonlinearity takes into account that the *material stiffness can change as the load increases*. For example, carbon steel is linear elastic until it reaches the yield point: the onset of nonlinearity. But many materials are never linear, such as the concrete stress–strain curves in EC2 **(Figure 4)**. As an assumption of a constant stiffness is useful in an analysis, there are two nominal stress-strain stiffnesses that we can extract from the curve:

→| **the secant stiffness:** the stiffness line to a stressed state (such as $0.4f_{cm}$ giving us E_{cm} in EC2). This gives us an effective stiffness when the stress is low.

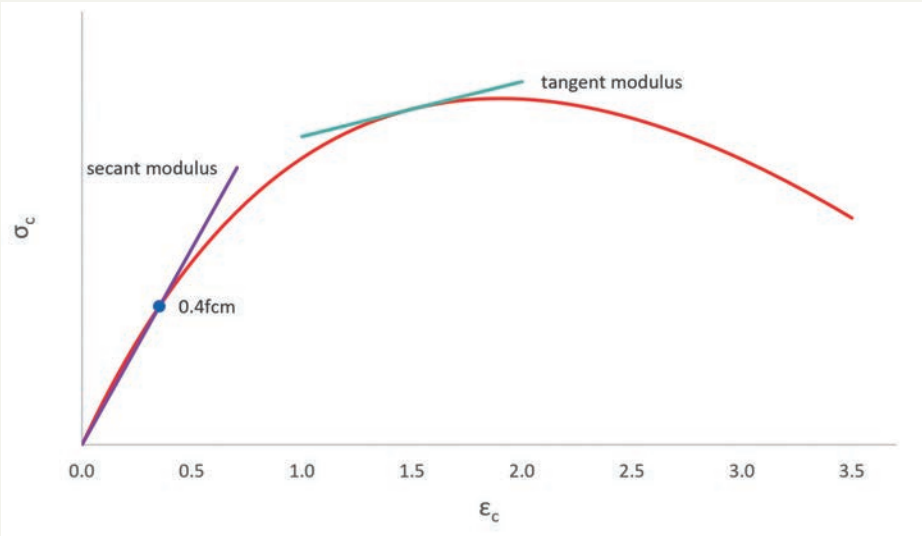
→| **the tangent stiffness:** the slope of the stiffness line at the stressed state. This gives the stiffness to match an incremental change in load.



↗**FIGURE 2:** Cable in an equilibrium position



↗**FIGURE 3:** Fabric and cable structure modelled in Oasys GSA[5]



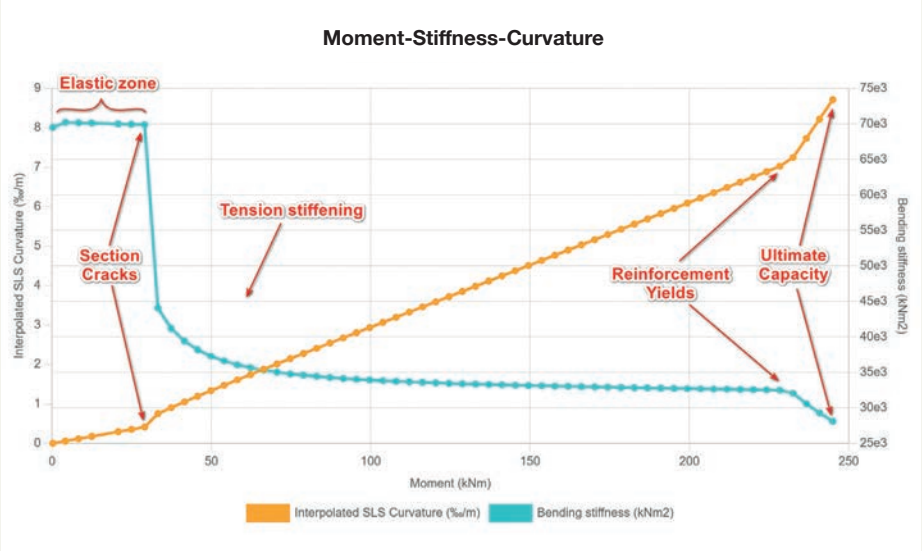
↗**FIGURE 4:** Stress–strain relation for concrete in compression EC2 eqn. 5.6[3]

* These loads can have the opposite effect on the cross-section stiffness.

† Impossible in reality due to self-weight, but a common starting point in a finite element analysis (FEA).

It can be easy to forget just how nonlinear reinforced concrete stiffness is. Its stiffness is dependent on the concrete material, the rebar material, the quantity and placement of both, plus the size of the moments and forces‡, the duration of the loads, and whether it has cracked the section or not. The resulting stiffness is thus highly variable across the range of its moment capacity **(Figure 5)**.

For a deeper diver into stress-strain relationships, the MIT training materials from David Roylance are recommended[7].



↗**FIGURE 5:** Oasys AdSec moment stiffness and curvature chart[6]

Directional/contact nonlinearity

Contact or directional nonlinearity means that the *structure works differently depending on which direction the load acts in*. This can be as simple as members that can carry only tension or only compression (or so little that we can safely ignore it). Tension-only cross-bracing and cables will be familiar examples. Other examples include the ability for a masonry wall to resist the wind thanks to its self-weight or for structural elements that resist sliding due to force-induced friction.

When a model has tension-only members, we or the FEA solver might ignore or deactivate those that go into compression; but we might need to consider their directional behaviour in more detail. For example, a long cable might start off very slack, with little initial stiffness, but as the tension increases, so does the stiffness.

Nonlinear analysis special considerations

Load factors and combinations

The first consideration needs special emphasis because it is too often forgotten: *the principle of superposition does not apply to nonlinear analysis!* You cannot analyse the individual loads and add the results together.

Contact nonlinearity can mean that structures such as masonry and suspension bridges are held in place by their dead load. Directional nonlinearity means that load paths change with load direction.

Material nonlinearity means that stresses do not increase at the same rate as strains or deflections, meaning that stiffness varies with load.

Geometric nonlinearity can radically change the shape of the structure, potentially increasing moments and redirecting forces.

All this means that doubling the load does not necessarily double the stress, strain, deflections or reactions, nor can you combine results. You must analyse the variable and permanent loads together, along with any factors, to ensure that the model is still stable and valid.

Modelling for buckling

To check for the possibility of geometric effects, the new Eurocodes require you to run modal buckling analyses, which are linear checks to find what multiples of the applied load reduce stiffnesses to zero somewhere in the model. Typically, if this factor is less than 10, then a nonlinear analysis is needed to find the more exact buckling behaviour plus any additional moments that are induced.

Buckling analyses, whether modal or nonlinear, need nodes at the locations where you want to check for buckling, such as the mid-height of a column. This is because p-delta and modal factors are calculated at the nodal positions. Note that some models can be sensitive to the mesh density so keep splitting the elements in half until the buckling load factors stop reducing significantly.

Section sizing

When designing the sections, remember that the moment distribution in an indeterminate frame is affected by the member stiffnesses. This means that *when you reanalyse after designing the sections, the members and other forces can change*. Thus, you need to redesign the sections, which then changes the moments, and so on. Know also that the sections you choose first can determine which sections you end up with. For more detail, see an *Introduction to engineering optimisation methods*[8], but remember that design is an iterative process and iterating even a linear one can lead to nonlinear effects.

It is also important to *design the sections or have at least a very good guess of their sizes before running any nonlinear analyses*. A buckling analysis, by definition, checks the sections you hope to build with, while a static nonlinear or p-delta can fail to converge if the sections are too small.

Convergence

A big difference between linear and nonlinear analyses is that of convergence. Unless you have built a mechanism, a linear analysis will usually give you an answer, even if it is not realistic§. Nonlinear models can consider second-order effects, buckling, plastic hinges, and so on, while searching for the equilibrium, but *if equilibrium is not possible with the model you have created, your analysis will fail to converge*. You must then work out why.

Struts can buckle, ties can yield and plastic hinges can remove stability. If this does happen in your model, try running a modal buckling analysis to understand the capacity or reducing the load by a large factor. If they do not help, then look for other problems in your model[9].

Connections

The behaviour of the connections can have a major effect on the behaviour of your model. With a linear analysis we might assume that a bolted connection is rotationally pinned because the expected movement will not exceed the rotational capacity of the connection (typically 2–3°). But *in nonlinear analysis you may need to model the actual stiffness of the connection*, perhaps with a moment-stiffness curve, or allow some freedom of movement before the connection locks up. This is all dependent on what question your model is addressing and what the capabilities of your program are.

Analysis methods

We don't have the space here to go into the details of the various nonlinear solvers available to you. Suffice to say that each has advantages and disadvantages, and that you can find more information in *Computational engineering*[10], NAFEMS guides[11], your program's documentation, etc.

‡ Compression can initially increase the stiffness of concrete cross-sections while tension can reduce it.

§ Models have been known to deflect several kilometres.

Analysis procedure

The updated *Eurocodes recommend you run several analyses*, which in general will be:

1) Linear elastic analysis
   a) Check member deflections
   b) Derive section sizes – repeat from #1 until consistent
2) Modal buckling analysis
   a) Check load factor
3) Nonlinear analysis (if frame deflections are large or modal load factors are small)
   a) Derive section sizes – repeat from #3 until consistent

Conclusion

While linear analysis is usually the right starting point, there are situations where nonlinear analyses are essential. These include stresses beyond the elastic limit, large deflections, slender members, lightweight structures, and iterative analyses.

Also, many of the linear assumptions, such as the principle of superposition, become invalid when using nonlinear analyses, so more care is needed to get useful results. Even where linear analysis is appropriate for the initial analysis and design, there are many situations where you must use nonlinear analyses or techniques: the wise engineer knows both when to use them and how to apply them correctly.

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Questions

To claim your CPD certificate, answer the questions online at **www.istructe.org/industry-cpd**. This module will close on 3 January 2027.

1) Which statement best explains why nonlinear analysis may be required?
a) It is always more accurate than linear analysis
b) It is needed when linear assumptions such as small deflections, constant stiffness or reversible loads no longer hold
c) It removes the need for engineering judgement
d) It is only required for dynamic analysis

2) Why does the principle of superposition not apply to nonlinear analysis?
a) Because nonlinear analysis cannot use load factors
b) Because nonlinear analysis only works with permanent loads
c) Because results from individual load cases may not combine in the same way once stiffness, geometry or contact conditions change
d) Because all nonlinear models are unstable

3) What is geometric nonlinearity primarily concerned with?
a) Changes in material stress-strain behaviour
b) Changes in load path or stiffness caused by displacement of the structure
c) Contact between separate parts of a model only
d) The use of smaller finite elements

4) In a modal buckling analysis, what does a low buckling load factor indicate?
a) The model is definitely incorrect
b) Linear analysis is sufficient and no further checks are needed
c) The structure may require nonlinear analysis to capture second-order effects more accurately
d) The structure has already yielded

5) Why can undersized members cause problems in nonlinear analysis?
a) They may yield, buckle or otherwise lose stiffness before the analysis reaches equilibrium
b) They make the mesh too fine
c) They prevent load factors from being applied
d) They make all results conservative

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